# TFY 4240 Løsning Øving11

Problem 1

**Problem 10.9**

(a) As in Ex. 10.2, for $t < r/c$, $\mathbf{A} = 0$; for $t > r/c$,

$$\mathbf{A}(r,t) = \left(\frac{\mu_0}{4\pi}\hat{\mathbf{z}}\right) 2 \int_0^{\sqrt{(ct)^2-r^2}} \frac{k(t-\sqrt{r^2+z^2}/c)}{\sqrt{r^2+z^2}}\,dz = \frac{\mu_0 k}{2\pi}\hat{\mathbf{z}}\left\{t\int_0^{\sqrt{(ct)^2-r^2}} \frac{dz}{\sqrt{r^2+z^2}} - \frac{1}{c}\int_0^{\sqrt{(ct)^2-r^2}} dz\right\}$$

$$= \left(\frac{\mu_0 k}{2\pi}\hat{\mathbf{z}}\right)\left[t\ln\left(\frac{ct+\sqrt{(ct)^2-r^2}}{r}\right) - \frac{1}{c}\sqrt{(ct)^2-r^2}\right].$$ Accordingly,

$$\mathbf{E}(r,t) = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 k}{2\pi}\hat{\mathbf{z}}\left\{\ln\left(\frac{ct+\sqrt{(ct)^2-r^2}}{r}\right) + t\left(\frac{r}{ct+\sqrt{(ct)^2-r^2}}\right)\left(\frac{1}{r}\right)\left(c+\frac{1}{2}\frac{2c^2t}{\sqrt{(ct)^2-r^2}}\right) - \frac{1}{2c}\frac{2c^2t}{\sqrt{(ct)^2-r^2}}\right\}$$

$$= -\frac{\mu_0 k}{2\pi}\hat{\mathbf{z}}\left\{\ln\left(\frac{ct+\sqrt{(ct)^2-r^2}}{r}\right) + \frac{ct}{\sqrt{(ct)^2-r^2}} - \frac{ct}{\sqrt{(ct)^2-r^2}}\right\}$$

$$= \boxed{-\frac{\mu_0 k}{2\pi}\ln\left(\frac{ct+\sqrt{(ct)^2-r^2}}{r}\right)\hat{\mathbf{z}}}$$ (or zero, for $t < r/c$).

$$\mathbf{B}(r,t) = -\frac{\partial A_z}{\partial r}\hat{\phi}$$

$$= -\frac{\mu_0 k}{2\pi}\left\{t\left(\frac{r}{ct+\sqrt{(ct)^2-r^2}}\right)\frac{\left[r\frac{1}{2}\frac{(-2r)}{\sqrt{(ct)^2-r^2}} - ct - \sqrt{(ct)^2-r^2}\right]}{r^2} - \frac{1}{2c}\frac{(-2r)}{\sqrt{(ct)^2-r^2}}\right\}\hat{\phi}$$

$$= -\frac{\mu_0 k}{2\pi}\left\{\frac{-ct^2}{r\sqrt{(ct)^2-r^2}} + \frac{r}{c\sqrt{(ct)^2-r^2}}\right\}\hat{\phi} = -\frac{\mu_0 k}{2\pi}\frac{(-c^2t^2+r^2)}{rc\sqrt{(ct)^2-r^2}}\hat{\phi} = \boxed{\frac{\mu_0 k}{2\pi r c}\sqrt{(ct)^2-r^2}\,\hat{\phi}.}$$

(b) $\mathbf{A}(r,t) = \frac{\mu_0}{4\pi}\hat{\mathbf{z}}\int_{-\infty}^{\infty}\frac{q_0\delta(t-\imath/c)}{\imath}\,dz$. But $\imath = \sqrt{r^2+z^2}$, so the integrand is even in $z$:

$$\mathbf{A}(r,t) = \left(\frac{\mu_0 q_0}{4\pi}\hat{\mathbf{z}}\right) 2\int_0^{\infty}\frac{\delta(t-\imath/c)}{\imath}\,dz.$$

Now $z = \sqrt{\imath^2 - r^2} \Rightarrow dz = \frac{1}{2}\frac{2\imath\,d\imath}{\sqrt{\imath^2-r^2}} = \frac{\imath\,d\imath}{\sqrt{\imath^2-r^2}}$, and $z = 0 \Rightarrow \imath = r$, $z = \infty \Rightarrow \imath = \infty$. So:

$$\mathbf{A}(r,t) = \frac{\mu_0 q_0}{2\pi}\hat{\mathbf{z}}\int_r^{\infty}\frac{1}{\imath}\delta\left(t-\frac{\imath}{c}\right)\frac{\imath\,d\imath}{\sqrt{\imath^2-r^2}}.$$

Now $\delta(t - \mathscr{r}/c) = c\delta(\mathscr{r} - ct)$ (Ex. 1.15); therefore $\mathbf{A} = \frac{\mu_0 q_0}{2\pi}\,\hat{\mathbf{z}}\,c\int_r^\infty \frac{\delta(\mathscr{r} - ct)}{\sqrt{\mathscr{r}^2 - r^2}}\,d\mathscr{r}$, so

$$\mathbf{A}(r,t) = \frac{\mu_0 q_0 c}{2\pi}\frac{1}{\sqrt{(ct)^2 - r^2}}\,\hat{\mathbf{z}} \quad \text{(or zero, if } ct < r\text{);}$$

$$\mathbf{E}(r,t) = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 q_0 c}{2\pi}\left(-\frac{1}{2}\right)\frac{2c^2 t}{[(ct)^2 - r^2]^{3/2}}\,\hat{\mathbf{z}} = \boxed{\frac{\mu_0 q_0 c^3 t}{2\pi[(ct)^2 - r^2]^{3/2}}\,\hat{\mathbf{z}}} \quad \text{(or zero, for } t < r/c\text{);}$$

$$\mathbf{B}(r,t) = -\frac{\partial \mathbf{A}_z}{\partial t}\,\hat{\boldsymbol{\phi}} = -\frac{\mu_0 q_0 c}{2\pi}\left(-\frac{1}{2}\right)\frac{-2r}{[(ct)^2 - r^2]^{3/2}}\,\hat{\boldsymbol{\phi}} = \boxed{\frac{-\mu_0 q_0 c r}{2\pi[(ct)^2 - r^2]^{3/2}}\,\hat{\boldsymbol{\phi}}} \quad \text{(or zero, for } t < r/c\text{).}$$

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**Problem 10.11**

In this case $\dot{\rho}(\mathbf{r},t) = \dot{\rho}(\mathbf{r},0)$ and $\dot{\mathbf{J}}(\mathbf{r},t) = 0$, so Eq. 10.29 $\Rightarrow$

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(\mathbf{r}',0) + \dot{\rho}(\mathbf{r}',0)t_r}{\mathscr{r}^2} + \frac{\dot{\rho}(\mathbf{r}',0)}{c\mathscr{r}}\right]\hat{\mathscr{r}}\,d\tau', \text{ but } t_r = t - \frac{\mathscr{r}}{c} \text{ (Eq. 10.18), so}$$

$$= \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(\mathbf{r}',0) + \dot{\rho}(\mathbf{r}',0)t}{\mathscr{r}^2} - \frac{\dot{\rho}(\mathbf{r}',0)(\mathscr{r}/c)}{\mathscr{r}^2} + \frac{\dot{\rho}(\mathbf{r}',0)}{c\mathscr{r}}\right]\hat{\mathscr{r}}\,d\tau' = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}',t)}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau'. \quad \text{qed}$$

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**Problem 10.12**

In this approximation we're dropping the higher derivatives of $\mathbf{J}$, so $\dot{\mathbf{J}}(t_r) = \dot{\mathbf{J}}(t)$, and Eq. 10.31 $\Rightarrow$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}^2}\left[\mathbf{J}(\mathbf{r}',t) + (t_r - t)\dot{\mathbf{J}}(\mathbf{r}',t) + \frac{\mathscr{r}}{c}\dot{\mathbf{J}}(\mathbf{r}',t)\right]\times\hat{\mathscr{r}}\,d\tau', \text{ but } t_r - t = -\frac{\mathscr{r}}{c} \text{ (Eq. 10.18), so}$$

$$= \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'. \quad \text{qed}$$

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