

Teoretisk fysikk I ^{24.}
15. august 1971
Løsning.

I

$$\underline{E} = \hat{E}_y E_0 \sin(kx - ct)$$

a)



$$Innholdslekket \bar{g}_x = \frac{1}{c^2} EH = \frac{\epsilon_0}{c} E^2$$

$$Middel \bar{\bar{g}}_x = \frac{\epsilon_0}{2c} E_0^2$$

Absorbert feltimpuls pr. overflatenhet = strålingsskoflu

$$= \bar{\bar{g}}_x \cdot c = \frac{\epsilon_0}{2} E_0^2$$

Dersom legges bøye seg med hastighet v , er strålingsskoflu til $\bar{\bar{g}}_x \cdot c - \bar{\bar{g}}_x \cdot v = \frac{\epsilon_0}{2} E_0^2 (1 - \beta)$

b)

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & -\frac{i}{c} E_x \\ -B_z & 0 & B_x & -\frac{i}{c} E_y \\ B_y & -B_x & 0 & \frac{i}{c} E_z \\ \frac{i}{c} E_x & \frac{i}{c} E_y & \frac{i}{c} E_z & 0 \end{pmatrix}$$

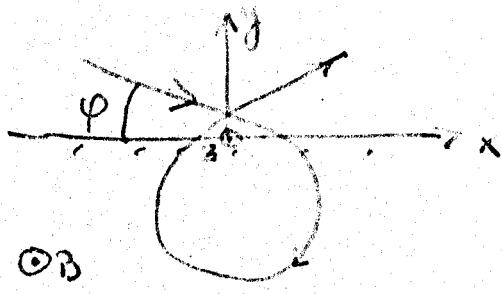
Da $B^2 - \frac{1}{c^2} E^2 = \frac{1}{2} \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu}$ følger det at uttrykket er relativistisk invariant.

Alternativt kan E^1, B^1 regnes ut i et annet referansestasjon K' ved hjelp av formelen $\tilde{F}_{\mu\nu}^1 = g_{\alpha\mu} g_{\beta\nu} F_{\alpha\beta}$. Dette gir

$$E_x^1 = E_x, E_y^1 = \gamma(E_y - v B_z), E_z^1 = \gamma(E_z + v B_y); (\gamma = (1 - \beta)^{-1})$$

$$B_x^1 = B_x, B_y^1 = \gamma(B_y + \frac{v}{c} E_z), B_z^1 = \gamma(B_z - \frac{v}{c} E_y).$$

$$\Rightarrow B^2 - \frac{1}{c^2} E^2 = B^1 \cdot \frac{1}{c^2} E^1 = \text{invariant.}$$



II. Partikelen befinder sig i magnetfeltet en del af en sirkel.

Når $Q > 0$ vil partikelbanen
→ del sirkel.

Radien R bestemmes ved

$$evB = \frac{mv^2}{R}; \quad R = \frac{mv}{eB} = \frac{mv\omega}{eB} \quad \left| \begin{array}{l} \text{Omloppstid} \\ T = \frac{2\pi R}{v} \end{array} \right.$$

$$\text{Jf xy-planet er } \{\hat{k} \times [(\hat{k} - \hat{p}) \times \dot{\hat{p}}]\}^2 = [(\hat{k} - \hat{p}) \times \dot{\hat{p}}]^2$$

$$= [\dot{p} \cos \theta - \dot{p} \beta]^2 = \dot{p}^2 (\cos \theta - \beta)^2$$

$$\text{b. } U(\hat{k}, t) \sim \frac{(\cos \theta - \beta)^2}{(1 - \cos \theta)^5} = 0 \text{ når } \cos \theta = \beta$$

Stillingen er symmetrisk om \hat{p} .

Stilling i z-retningen: I dette tilfællet er

$$\hat{k} \times [(\hat{k} - \hat{p}) \times \dot{\hat{p}}] = (\hat{k} - \hat{p})(\hat{k} \cdot \dot{\hat{p}}) - \dot{\hat{p}}(\hat{k} \cdot (\hat{k} - \hat{p})) = -\dot{\hat{p}}$$

$$\text{Da } |\dot{\hat{p}}| = \frac{v^2}{R} \text{ er } \dot{\hat{p}}^2 = \frac{1}{c^2} \left(\frac{v^2}{R}\right)^2$$

$$U(\hat{e}_z) = U(\hat{e}_z, t) \cdot T = \frac{1}{4\pi} \frac{ce^2}{4\pi} \beta \cdot \frac{2\pi R}{v} = \frac{\mu_0 e v}{8\pi c} \cdot \frac{1}{R} = \frac{\mu_0 e B}{8\pi c m}$$

$v \ll c$: Stillingens andel:

$$\begin{aligned} A &= \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{c^2}{c^3} \int_0^T \ddot{U} \cdot \dot{U} dt = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{c^2}{c^3} \left[\int_0^T \ddot{U} \cdot \dot{U} - \int_0^T \dot{U}^2 dt \right] \\ &= -\frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{c^2}{c^3} \dot{U}^2 \cdot T = -\frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{c^2}{c^3} \frac{v^4}{R^2} \cdot \frac{2\pi R}{v} = -\frac{e^2 v^3}{3\epsilon_0 c^3} \cdot \frac{1}{R} \\ &= -\frac{e^2 v^3 B}{3\epsilon_0 c^3 m} = \div \text{ den totale udstille energi.} \end{aligned}$$