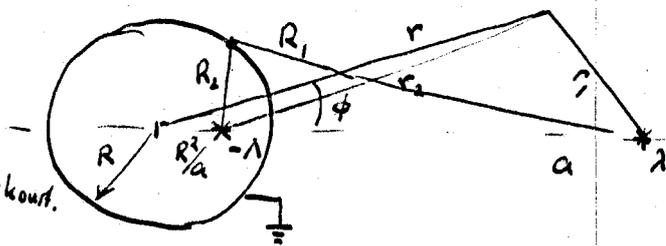


Lösningar.

Ia) Potentialet från λ i a
 λ i R^2/a är

$$\varphi(r, \phi, z) = -\frac{\lambda}{2\pi\epsilon_0} (\lambda \ln r_1 - \lambda \ln r_2) + \text{konst.}$$



λ bestämt av

$$\varphi(R, \phi, z) = -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_1}{R_2} + \text{konst.} = \text{konst. uavhängig av } \phi \text{ (och } z)$$

$$\frac{R_1}{R_2} = \frac{[(a - R \cos \phi)^2 + (R \sin \phi)^2]^{1/2}}{[(\frac{R^2}{a} - R \cos \phi)^2 + (R \sin \phi)^2]^{1/2}} = \frac{[a^2 - 2aR \cos \phi + R^2]^{1/2}}{[\frac{R^4}{a^2} - 2\frac{R^3}{a} \cos \phi + R^2]^{1/2}}$$

$$= \left(\frac{a}{R}\right)^{\lambda} [a^2 - 2aR \cos \phi + R^2]^{\lambda - \lambda} = \text{konst.} \quad \text{hvis } \lambda = \lambda$$

Med normeringskonstanten valgt slikt att $\varphi = 0$ på cylinderflaten:

$$\varphi(r, \phi, z) = -\frac{\lambda}{4\pi\epsilon_0} \ln \frac{(a - r \cos \phi)^2 + (r \sin \phi)^2}{(\frac{R^2}{a} - r \cos \phi)^2 + (r \sin \phi)^2} \cdot \left(\frac{R}{a}\right)^{\lambda}$$

Eller med komplex vektor $\xi = r e^{i\phi}$

$$\varphi(\xi, z) = -\frac{\lambda}{2\pi\epsilon_0} \ln \left(\frac{|\xi - a|}{|\xi - \frac{R^2}{a}|} \left| \frac{R}{a} \right| \right)$$

Fältet bestäms av

$$\vec{E}(r, \phi, z) = -\nabla\varphi = \left(-\frac{\partial\varphi}{\partial r}, -\frac{1}{r} \frac{\partial\varphi}{\partial\phi}, 0 \right)$$

$$= \left(\frac{\lambda}{2\pi\epsilon_0} \left(\frac{r - a \cos \phi}{r^2} - \frac{r - \frac{R^2}{a} \cos \phi}{r^2} \right), \frac{\lambda}{2\pi\epsilon_0} \left(\frac{a \sin \phi}{r^2} - \frac{\frac{R^2}{a} \sin \phi}{r^2} \right), 0 \right)$$

eller med kartesiska koordinater ($r \cos \phi = x$ $r \sin \phi = y$)

$$\vec{E}(x, y, z) = \left(\frac{\lambda}{2\pi\epsilon_0} \left(\frac{a-x}{r^2} - \frac{\frac{R^2}{a}-x}{r^2} \right), \frac{\lambda}{2\pi\epsilon_0} \left(\frac{y}{r^2} - \frac{y}{r^2} \right), 0 \right)$$

eller med komplex vektor

$$\vec{E}(\xi, z) = \frac{\lambda}{2\pi\epsilon_0} \left(\frac{\xi - a}{|\xi - a|^2} - \frac{\xi - \frac{R^2}{a}}{|\xi - \frac{R^2}{a}|^2} \right)$$

$$\text{I b) } \nabla^2 \varphi = -\rho/\epsilon_0 = \begin{cases} \frac{\Lambda}{2\pi} \frac{1}{r} \frac{\partial}{\partial r} r = 0 & r > R \\ -\frac{\Lambda}{4\pi} \frac{1}{r} \frac{\partial}{\partial r} \left(-\frac{2r^2}{R^2}\right) = \frac{\Lambda}{\pi R^2} & r \leq R \end{cases}$$

Ingen flate ladning i $r = R$ da kontinuerlig

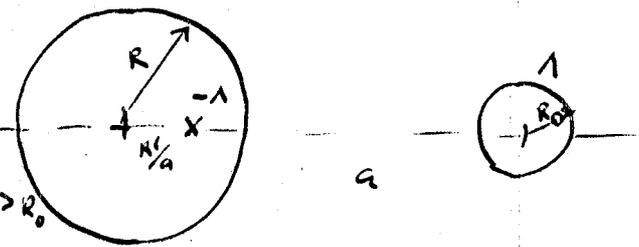
$$\varphi(R) = 0 \quad \frac{\partial \varphi}{\partial r} = \begin{cases} -\frac{\Lambda}{2\pi\epsilon_0} \frac{1}{R} & \text{for } r \rightarrow R+\delta \\ \frac{\Lambda}{4\pi\epsilon_0} \left(-\frac{2R}{R^2}\right) = -\frac{\Lambda}{2\pi\epsilon_0} \frac{1}{R} & r \rightarrow R-\delta \end{cases}$$

Total ladning innenfor $r < R$ $\int_0^{2\pi} \int_0^R \frac{\Lambda}{\pi R^2} r dr d\varphi = \underline{\Lambda}$ Stemmer.

II c)

Ekvivalent med

$$\Lambda \text{ i } a \text{ og } -\Lambda \text{ i } \frac{R^2}{a}$$

$$\varphi(\xi, z) = \begin{cases} -\frac{\Lambda}{2\pi\epsilon_0} \ln\left(\frac{|\xi-a|}{|\xi-\frac{R^2}{a}|} \frac{R}{a}\right) & \text{utenfor cylinderene} \\ & |\xi| > R \text{ og } |\xi-a| > R_0 \\ -\frac{\Lambda}{2\pi\epsilon_0} \left\{ \frac{1}{2} \left(\left| \frac{\xi-a}{R_0} \right|^2 - 1 \right) - \ln \frac{|\xi-\frac{R^2}{a}| a}{R_0 R} \right\} & \text{innenfor cylinderladningene} \\ & |\xi-a| < R_0 \end{cases}$$


Tiltrekkende kraft

$$F = (-\Lambda) \frac{-\Lambda}{2\pi\epsilon_0 \left(a - \frac{R^2}{a}\right)} = \underline{\underline{\frac{\Lambda^2 a}{2\pi\epsilon_0 (a^2 - R^2)}}}$$

TF IC 14.8.72 Lörninger.

$$\begin{aligned} \text{II a)} \quad \nabla \times \vec{E} &= - \frac{\partial \vec{B}}{\partial t} & - \vec{e} \\ \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} + \vec{j} & \vec{e} \end{aligned}$$

$$-\vec{e}(\nabla \times \vec{E}) + \vec{E}(\nabla \times \vec{H}) = \vec{e} \frac{\partial \vec{D}}{\partial t} + \vec{E} \frac{\partial \vec{D}}{\partial t} + \vec{E} \vec{j}$$

med $\vec{B} = \mu \vec{H}$ $\vec{D} = \epsilon \vec{E}$

$$-\nabla(\vec{E} \times \vec{H}) = \frac{\partial}{\partial t} \frac{1}{2}(\epsilon \vec{D} + \mu \vec{H}) + \vec{E} \vec{j}$$

$$\underline{u = \frac{1}{2}(\epsilon E^2 + \mu H^2)} \quad \underline{S = \vec{E} \times \vec{H}}$$

b) Energi i lyrbunt med tvärsnitt A

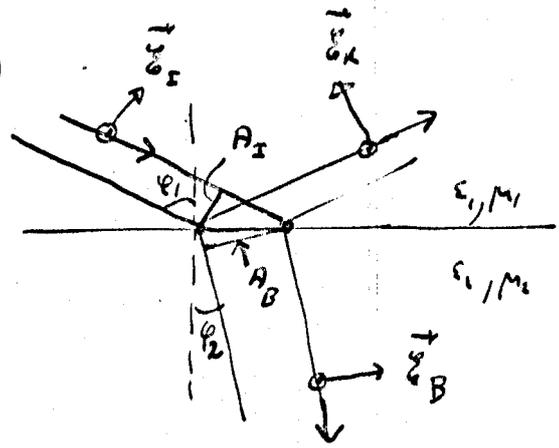
$$S \cdot A = |\vec{E} \times \vec{H}| A = \sqrt{\frac{\epsilon}{\mu}} E^2 \cdot A$$

Energi bevarelse:

$$\sqrt{\frac{\epsilon_1}{\mu_1}} E_I^2 A_I = \sqrt{\frac{\epsilon_1}{\mu_1}} E_R^2 A_I + \sqrt{\frac{\epsilon_2}{\mu_2}} E_B^2 A_B$$

eller

$$E_I^2 - E_R^2 = \sqrt{\frac{\epsilon_2 \mu_1}{\epsilon_1 \mu_2}} \frac{A_B}{A_I} E_B^2$$



Tvärsnittets förändring $\frac{A_B}{A_I} = \frac{\cos \phi_2}{\cos \phi_1}$

Gränsebetingelse:

E_{tang} = kontinuerlig: $(E_I - E_R) \cos \phi_1 = E_B \cos \phi_2$

H_{tang} = kontinuerlig: $H_I + H_R = H_B \Rightarrow \sqrt{\frac{\epsilon_1}{\mu_1}} (E_I + E_R) = \sqrt{\frac{\epsilon_2}{\mu_2}} E_B$

Ventre side blir:

$$E_I^2 - E_R^2 = (E_I + E_R)(E_I - E_R) = \sqrt{\frac{\epsilon_2 \mu_1}{\epsilon_1 \mu_2}} E_B \frac{\cos \phi_2}{\cos \phi_1} E_B$$

Höjre side blir det samma när A_B/A_I sätter in.

TF IC 14.8.72 Lösningar.

III a) $k(\omega) = \frac{1}{R - i\omega L} = \frac{R}{R^2 + \omega^2 L^2} + \frac{i\omega L}{R^2 + \omega^2 L^2}$

$$\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{xL}{R^2 + x^2 L^2} \frac{1}{x - \omega} dx = \frac{1}{\pi} \frac{1}{2} \left(\int_{-\infty}^{\omega} + \int_{\omega}^{\infty} \right) \frac{x}{L(x + i\frac{R}{L})(x - i\frac{R}{L})(x - \omega)} dx$$

$$= \frac{1}{2\pi} 2\pi i \left[\frac{i\frac{R}{L} \cdot 2}{L \cdot 2i\frac{R}{L} (i\frac{R}{L} - \omega)} + \frac{\omega}{L(\omega^2 + \frac{R^2}{L^2})} \right] = \frac{R}{\omega^2 L^2 + R^2} = k'(\omega)$$

$$\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{R}{R^2 + x^2 L^2} \frac{1}{x - \omega} dx = \frac{1}{\pi} i \left[\frac{2}{L \cdot 2i\frac{R}{L} (i\frac{R}{L} - \omega)} + \frac{1}{L(\omega^2 + \frac{R^2}{L^2})} \right] = -\frac{\omega L}{\omega^2 L^2 + R^2} = -k''(\omega)$$

b) Här $k''(\omega_0) \neq 0$ er

$$k'(\omega_0) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{k''(x)}{x - \omega_0} dx \approx \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{k''(x)}{x - \omega_0} dx$$

Omkring $\omega_0 > 0$ er da nær vi benyttes $k''(-\omega) = -k''(\omega)$

$$\frac{dk'}{d\omega} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{k''(x)}{(x - \omega)^2} dx = \frac{1}{\pi} \int_0^{\infty} \left(\frac{k''(x)}{(x - \omega)^2} + \frac{k''(-x)}{(-x - \omega)^2} \right) dx$$

$$= \frac{1}{\pi} \int_0^{\infty} k''(x) \left(\frac{1}{(x - \omega)^2} - \frac{1}{(x + \omega)^2} \right) dx > 0 \quad \text{da integranden er positiv for } \omega > 0 \text{ og } x > 0$$