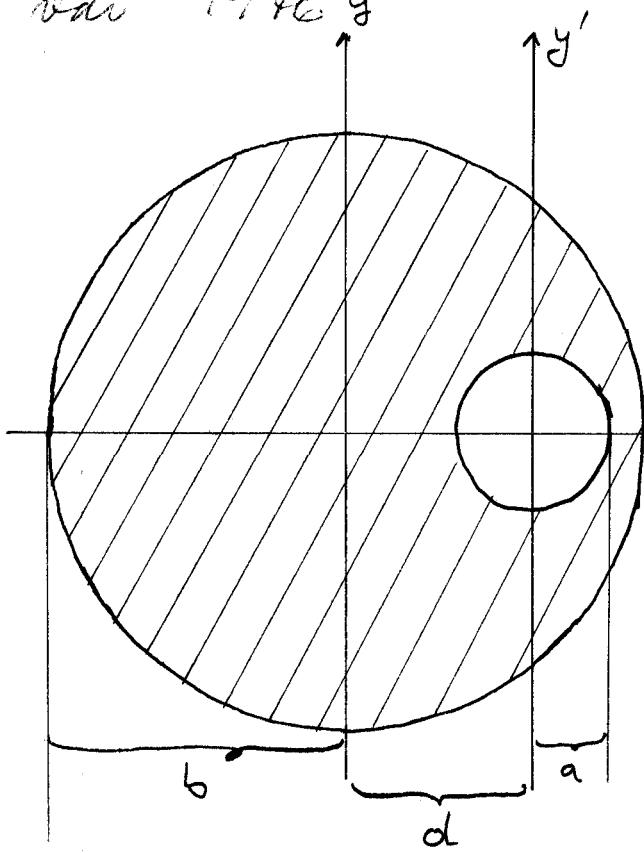


Fag 24516 Test oppgave 1C  
Vær 1970 g

Løsning

①

Oppgave 1



Ved superposisjonsprinsippet

$$\vec{B}(\vec{r}) = \vec{B}_b(\vec{r}) - \vec{B}_a(\vec{r})$$

der  $\vec{B}_b$ ,  $\vec{B}_a$  er  
de feltet som dannes  
ved konstant strøm

$\vec{j} = j \vec{e}_z$  langs massive  
sylinder med radii  $b, a$   
sentert som vist i figuren.  
(z aksem står  $\perp$  på figur).

No symmetrigrunnar  $\vec{B}_b \perp \vec{e}_z$ ,  $\vec{B}_a \perp \vec{e}_z$  og

$$\vec{B}_b(\vec{r}) = B_b(r) \vec{e}_\varphi \quad ; \quad \vec{B}_a(\vec{r}') = B_a(r') \vec{e}'_\varphi,$$

der  $\vec{e}_\varphi = (-y \vec{e}_x + x \vec{e}_y)/r$  (cylindisk koordinat).

Med Maxwells likning  $\int_C \vec{B} \cdot d\vec{r} = \mu_0 \int_C \vec{j} \cdot d\vec{l}$

for ved å ta  $C =$  sirkel med radius  $r$ :

$$\vec{B}_b(r) = \begin{cases} \frac{\mu_0 j}{2} [-y \vec{e}_x + x \vec{e}_y] & \text{for } r \leq b \\ \frac{\mu_0 j b^2}{2 r^2} [-y \vec{e}_x + x \vec{e}_y] & \text{for } b \leq r \end{cases}$$

$$\vec{B}_a(r') = \begin{cases} \frac{\mu_0 j}{2} [-y' \vec{e}_x + x' \vec{e}_y] & \text{for } r' \leq a \\ \frac{\mu_0 j a^2}{2 r'^2} [-y' \vec{e}_x + x' \vec{e}_y] & \text{for } r' \geq a \end{cases}$$

(2)

Med  $x' = x - d$ ,  $y' = y$  fås

1) I hulrommet:

$$\vec{B}(\vec{r}) = \frac{\mu_0 j}{2} d \vec{e}_y \quad (\text{homogenfelt i } y\text{-retning})$$

2) I lederen:

$$\vec{B}(\vec{r}) = \frac{\mu_0 j (r'^2 - a^2)}{2r'} \vec{e}'_x + \frac{\mu_0 j}{2} d \vec{e}'_y$$

(Obs: enkelt i koordinatsystem  $x', y', z'$ ).

3) I vakuum utenfor sylinderen:

$$\begin{aligned} \vec{B}(\vec{r}) = & \frac{\mu_0 j}{2} \left\{ -y \left[ \frac{b^2}{r^2} - \frac{a^2}{(\vec{r} - d \vec{e}_x)^2} \right] \vec{e}_x \right. \\ & \left. + \left[ \frac{b^2}{r} - \frac{a^2(x-d)}{(\vec{r} - d \vec{e}_x)^2} \right] \vec{e}_y \right\} \end{aligned}$$

(3)

## Übung 2

1)  $r > R$

$$\begin{aligned}\varphi(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int_K dF \frac{\sigma(\vec{r}')}{|\vec{r}-\vec{r}'|} \\ &= \frac{1}{\epsilon_0} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{Q_{\ell m}}{2\ell+1} \frac{Y_{\ell m}(\theta, \varphi)}{r^{\ell+1}}\end{aligned}$$

oder (wod  $\rho(\vec{r}') = \delta(R-r') \sigma(\vec{r}')$ ,  $\Omega = (\vartheta, \varphi)$ ):

$$\begin{aligned}Q_{\ell m} &= \int d^3 r' \rho(\vec{r}') r'^{\ell} Y_{\ell m}^*(\Omega') \\ &= \sigma_0 R^{\ell+2} \int d\Omega' \cos \vartheta' Y_{\ell m}^*(\Omega') \\ &= \sqrt{\frac{4\pi}{3}} R^{\ell+2} \sigma_0 \int d\Omega' Y_{\ell 0}(\Omega') Y_{\ell m}^*(\Omega') \\ &= \sqrt{\frac{4\pi}{3}} R^3 \sigma_0 \delta_{\ell 1} \delta_{m 0}\end{aligned}$$

Nun für

$$\varphi(\vec{r}) = \frac{R^3 \sigma_0}{3\epsilon_0} \frac{\cos \vartheta}{r^2} \quad (\text{et dipolfeld : dipolmoment}$$

$$\text{er } \vec{P} = \frac{4\pi}{3} R^3 \sigma_0 \vec{e}_z).$$

(4)

$$2) \quad r < R$$

Analog

$$\varphi(\vec{r}) = \frac{\sigma_0}{\epsilon_0} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{r^\ell}{(2\ell+1)R^{\ell+1}} Y_{\ell m}(\Omega) \int d\Omega' \cos \theta' Y_{\ell m}^*(\Omega')$$

$$= \frac{\sigma_0}{3\epsilon_0} r \cos \theta$$

—————

Oppgave 3

Standard