

Liongur

$\mathbf{E}, \mathbf{a}, \mathbf{b}, \mathbf{c}$ se forstørre.

$$\nabla^2 \Phi = 0$$

$$\text{Fullständig sett: } \theta \propto r \quad \Phi(r, \theta, \varphi) = \sum b_{lm}(r) P_l^m(\cos \theta) e^{im\varphi}$$

$$\text{Rotationssym-} \quad \frac{\partial \Phi}{\partial \varphi} = 0 \quad \text{betr. } m=0 \quad b_{l0} = 0 \quad \text{for } \omega \neq 0$$

$$\nabla^2 \Phi = \sum_l \left(\frac{\partial^2}{\partial r^2} b_l + \frac{2}{r} \frac{\partial}{\partial r} b_l - \frac{l(l+1)}{r^2} b_l \right)$$

$$b_l = \sum a_n r^n \quad \text{gir} \quad a_n [n(n-1) + 2n - l(l+1)] = 0 \quad n = \begin{cases} l \\ -(l+1) \end{cases}$$

$$\underline{b_l = \frac{A_l}{r^{l+1}} + B_l r^l}$$

$$b) \text{ Utvifte kuler: } \Phi^{ul} = \sum_l \left(\frac{A_l}{r^{l+1}} + B_l r^l \right) P_l(\cos \theta)$$

$$\text{Inne: } b_{l0} \quad \Phi^{inn} = \sum_l C_l r^l P_l(\cos \theta) \quad \text{da etter divergent: } r^l.$$

$$\text{Grennhet: } r \rightarrow \infty \quad \Phi \rightarrow -B_0 z \quad \text{gir} \quad \underline{B_0 = -B_0}, \quad \underline{B_l = 0} \quad \text{for } l \neq 1$$

$$r=R \quad \underline{C_{l0}^{ul} = C_{l0}^{inn}} \quad \underline{-\frac{1}{R} \frac{\partial \Phi^{ul}}{\partial r} \Big|_{r=R} = -\frac{1}{R} \frac{\partial \Phi^{inn}}{\partial r} \Big|_{r=R}}$$

$$-\sum_l C_l R^{l-1} P_l' = -\sum_l \left(\frac{A_l}{R^{l+2}} + B_l R^{l-1} \right) P_l'$$

$$\underline{C_l = B_l + \frac{A_l}{R^{2l+1}}}$$

$$\underline{D_{norm}^{ul} = D_{norm}^{inn}} \quad -\epsilon_0 \frac{1}{R} \frac{\partial \Phi^{ul}}{\partial r} \Big|_{r=R} = -\epsilon_0 \frac{1}{R} \left(\frac{\partial \Phi^{inn}}{\partial r} \right)_{r=R}$$

$$-\epsilon_0 \sum_l ((l+1) \frac{A_l}{R^{l+3}} + l B_l R^{l-2}) P_l = -\sum_l l \epsilon_0 R^{l-2} P_l$$

$$\underline{\epsilon_0 l C_l = l B_l - (l+1) \frac{A_l}{R^{2l+1}}}$$

$$\text{Som vist gir: } A_l = C_l = 0 \quad \text{for alle } l \neq 1 \quad \text{når } B_l = -B_0, \quad B_0 \neq 0 \quad (l \neq 1)$$

$$\underline{C_1 = -\frac{3}{2+\epsilon_r} B_0} \quad \underline{A_1 = \frac{\epsilon_r-1}{\epsilon_r+2} R^3 B_0}$$

$$\underline{\Phi^{ul} = -\left(\frac{3}{\epsilon_r+2}\right) B_0 r \cos \theta}$$

$$\underline{\Phi^{ul} = -B_0 r \cos \theta + \frac{\epsilon_r-1}{\epsilon_r+2} B_0 \frac{R^3}{r^2} \cos \theta}$$

$$\text{III a)} \quad \operatorname{div} \vec{j} + \frac{\partial \rho}{\partial t} = 0 \quad \Rightarrow \quad \operatorname{div} j_w(\vec{r}) - i\omega_0 \rho_w(\vec{r}) = 0$$

$$\text{b)} \quad \vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}_w(\vec{r}') e^{i\omega_0 \frac{|\vec{r}-\vec{r}'|}{c}}}{|\vec{r}-\vec{r}'|} d^3 r' e^{-i\omega_0 t} = \vec{A}_w(\vec{r}) e^{-i\omega_0 t}$$

$$\Phi(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho_w(\vec{r}') e^{i\omega_0 \frac{|\vec{r}-\vec{r}'|}{c}}}{|\vec{r}-\vec{r}'|} d^3 r' e^{-i\omega_0 t} = \Phi_w(\vec{r}) e^{-i\omega_0 t}$$

För stora avstånden

$$|\vec{r}-\vec{r}'| = \sqrt{r^2 - 2\vec{r}\cdot\vec{r}'+r'^2} = r \left[1 - \frac{\vec{r}\cdot\vec{r}'}{r} + O((\frac{r'}{r})^2) \right] = r - \frac{\vec{r}\cdot\vec{r}'}{r} +$$

$$\vec{A}_w(\vec{r}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{j}_w(\vec{r}') e^{-i\vec{k}\cdot\vec{r}'} d^3 r' \quad \text{med} \quad \vec{k} = \frac{\omega_0 \vec{r}}{c} = \frac{\omega_0}{c} \hat{r}$$

$$\text{c)} \quad B_w(\vec{r}) = \operatorname{curl} \vec{A}_w(\vec{r}) \quad \text{da} \quad e^{-i\omega_0 t} \quad \text{bare er feller faktor}$$

$$= \nabla \times \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{j}_w(\vec{r}') e^{-i\vec{k}\cdot\vec{r}'} d^3 r'$$

$$= \left(ik - \frac{\hat{r}}{r} \right) \times \vec{A}_w = \frac{i[\vec{k} \times \vec{A}_w]}{r} \quad \text{när} \quad r \rightarrow \infty$$

$$\vec{E}(\vec{r}, t) = -\operatorname{grad} \Phi - \frac{\partial \vec{A}}{\partial t} = (-\operatorname{grad} \Phi_w + i\omega_0 \vec{A}_w) e^{-i\omega_0 t}$$

$$\vec{E}_w(\vec{r}) = i\omega_0 \vec{A}_w - ik \Phi_w$$

$$\text{Lorentz konvention:} \quad \operatorname{div} \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0 \Rightarrow ik \vec{A}_w - \frac{1}{c} i\omega_0 \Phi_w = 0$$

$$\Phi_w = \frac{ik \vec{A}_w c}{\omega_0} = c \vec{k} \cdot \vec{A}_w$$

$$\vec{E}_w(\vec{r}) = i\omega_0 \left(\vec{A}_w - \vec{k} (\vec{k} \cdot \vec{A}_w) \right) = \frac{i\omega_0 [\vec{k} \times \vec{A}_w] \times \vec{k}}{c}$$

Poyntingvektorn: Tänk nu att lösa $\vec{E} = \frac{1}{c} \operatorname{curl} \vec{B}_w e^{-i\omega_0 t}$

$$\int S_{\perp} d\vec{r} = \int [\vec{E}_w(\vec{r}) \times \vec{B}_w(\vec{r})] e^{i(\omega_0 + \omega'_0)t} d\omega d\omega' dt = \int [\vec{E}_w(\vec{r}) \times \vec{B}_{w'}(\vec{r})] 2\pi \delta(\omega_0 + \omega') d\omega d\omega'$$

$$= 2\pi \int [\vec{E}_w(\vec{r}) \times \vec{B}_{-w}(\vec{r})] d\omega = 2\pi \int [\vec{E}_w(\vec{r}) \times \vec{B}_w^*(\vec{r})] d\omega$$

$$= \frac{2\pi}{\mu_0} \int i\omega_0 [\vec{k} \times \vec{A}_w] \times \vec{k} \times i\omega_0 [\vec{k} \times \vec{A}_w^*] d\omega$$

$$= \frac{2\pi \omega_0^2}{\mu_0 c} A |\vec{k} \times \vec{A}_w|^2 d\omega$$

$$\vec{r}_w = \frac{2\pi \omega_0}{\mu_0 c} |\vec{k} \times \vec{A}_w|^2$$

Lösungssatz

III d) Fourier-Optik: für einen Strom beobachtet:

$$\vec{I} = \hat{\epsilon}_\phi I_0 \sin \omega t = \frac{1}{2} I_0 \hat{\epsilon}_\phi (e^{i\omega t} + e^{-i\omega t})$$

$$= \int \int_{\text{Luft}} dr dz (e^{i\omega t} + e^{-i\omega t}) = \int \hat{\epsilon}_\phi \frac{I_0}{2} \delta(r) \delta(z) dr dz (e^{i\omega t} + e^{-i\omega t})$$

Kontinuitätsgesetze gelten für Ladungsdichten

$$i\omega g_w = \operatorname{div} \vec{j}_w = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{I_0}{2} \delta(r) \delta(z) \right) = 0 \Rightarrow g_w = 0 \text{ für } \omega \neq 0$$

Heraus $\vec{D}_w^{(0)} = 0$

$$\vec{D}_w^{(mag)} = \frac{1}{2} \int [\hat{\epsilon}_r \times \hat{\epsilon}_\phi] r \frac{I_0}{2} \delta(z) \delta(r) dr dz r d\phi = \hat{\epsilon}_z \frac{I_0}{2} \pi R^2$$

$$\text{e)} P(\omega, \theta) = \frac{\mu_0 \omega^2}{8\pi^2 c} \left(\frac{\omega}{c} \right)^2 |D_w^{(mag)}|^2 / [n \times [\hat{\epsilon}_z \times \hat{n}]]^2 = \frac{\mu_0 \omega^4}{8\pi^2 c^3} |D_w^{(mag)}|^2 \sin^2 \theta$$

$$= \frac{\mu_0}{32\pi^2} \frac{\omega^4}{c^3} I_0 (\pi R^2)^2 \sin^2 \theta$$

$$\underline{P(\omega)} = \frac{\mu_0}{12\pi} (\pi R^2)^2 I_0^2 \frac{\omega^4}{c^3}$$

$$\int \sin^2 \theta d(\sin \theta) d\phi = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$