UNIVERSITETET I TRONDHEIM
NORGES TEKNISKE HØGSKOLE
INSTITUTT FOR TEORETISK FYSIKK

Faglig kontakt under eksamen:
F.aman.Finn Bakke
Tlf. 3649

Kontinuasjonseksamen i fag 71516 ELEKTROMAGNETISK TEORI

Lørdag 21.august 1982

kl.0900-1500

Tillatte hjelpemidler: Otto Øgrim: Størrelser og enheter i fysikken
K.Rottmann: Mathematische Formelsammlung
Regnestav/lommekalkulator.

## Problem 1

In this problem we consider electromagnetic radiation in a medium with a dielectric constant $\varepsilon$ , a magnetic permeability $\mu$ and a zero conductivity, $\sigma=0$ . Both $\varepsilon$ and $\mu$ are constant and real. The radiation field has the following form

$$\vec{E}(\vec{r},t) = \vec{E}_0 \exp(i\vec{k}\cdot\vec{r}-i\omega t)$$

$$\vec{H}(\vec{r},t) = \vec{H}_0 \exp(i\vec{k}\cdot\vec{r}-i\omega t)$$

a) Derive: $\vec{k}\cdot\vec{E}_0 = \vec{k}\cdot\vec{H}_0 = 0$

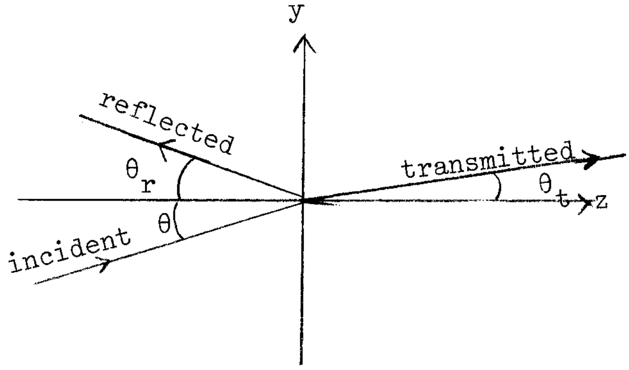
b) Derive: $\vec{H}_0 = \frac{c}{\mu\omega}\vec{k}\wedge\vec{E}_0$

c) Derive the dispersion relation $k(\omega) = \frac{\omega}{c}\sqrt{\varepsilon\mu}$

Note that $k(\omega)$ is real.

d) What is the most important difference if the conductivity is finite and how does this effect the intensity of the beam as a function of position (no derivation needed,only give a short comment).

We now consider the reflection and transmission of light at a flat interface between two non conductive media with dielectric constants $\varepsilon_1$ and $\varepsilon_2$ and magnetic permeabilities $\mu_1$ and $\mu_2$ .




The interface is chosen along the x-y plane. For $z<0$ the field is the sum of the incident and the reflected radiation field. For $z>0$ it is equal to the transmitted field. For the electric field one therefore has

$$\vec{E}(\vec{r},t)=\begin{cases}\vec{E}_i\exp(i\vec{k}_i\cdot\vec{r}-i\omega t)+\vec{E}_r\exp(i\vec{k}_r\cdot\vec{r}-i\omega t) & \text{for } z<0\\ \vec{E}_t\exp(i\vec{k}_t\cdot\vec{r}-i\omega t) & \text{for } z>0\end{cases}$$

and similarly for the magnetic field. The properties derived in a, b and c are now true for the incident, the reflected and the transmitted fields respectively.

e) Give these properties explicitly.

f) What are the boundary conditions at $z=0$ (No derivation).

g) Show that $\theta_r=\theta_i$ and $n_1\sin\theta_i=n_2\sin\theta_t$ where the refractive indices are defined as $n_1\equiv\sqrt{\varepsilon_1\mu_1}$ and $n_2\equiv\sqrt{\varepsilon_2\mu_2}$ .
(The angles of incidence, reflection and transmission are related to the wave vectors by:

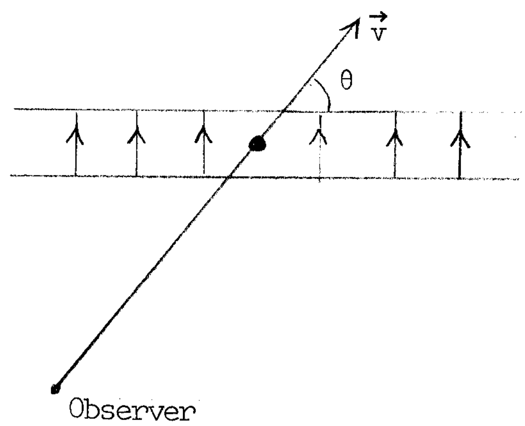
$$\vec{k}_i=|\vec{k}_i|(0,\sin\theta_i,\ \cos\theta_i)$$
$$\vec{k}_r=|\vec{k}_r|(0,\sin\theta_r,-\cos\theta_r)$$
$$\vec{k}_t=|\vec{k}_t|(0,\sin\theta_t,\cos\theta_t))\ .$$

Bonus question

h) Derive expressions for $\vec{E}_t$ and $\vec{E}_r$ if $\vec{E}_i=(E_i,0,0)$ .

Problem 2

A plan parallel condensator has a velocity $v$ with respect to the observer. The surface area of the plates is $S$ . The distance between the plates is $d$ . The electric field $E$ is homogeneous and orthogonal on the plates (effects at the edge are



neglected) inside the condensator and zero outside the condensator.

a) Give the electromagnetic energy density as well as the total electromagnetic energy in the rest frame of the condensator.

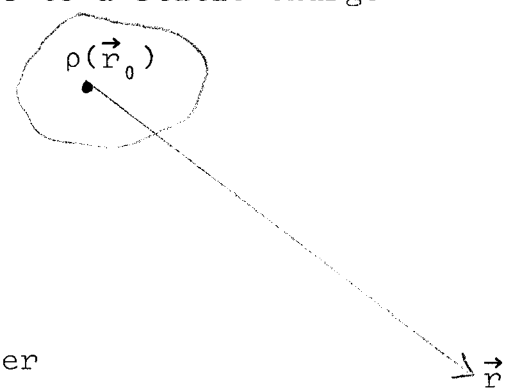
b) Give the electric and the magnetic fields in the condensator in the frame of the observer (choose the x axis such that $\vec{v}=(v,0,0)$ and take the electric field in the rest frame of the condensator as $\vec{E}=E(\sin\theta, \cos\theta,0)$).

c) Give the electromagnetic energy and the momentum density in the frame of the observer.

d) Calculate the total electromagnetic energy and momentum of the condensator in the frame of the observer.

Problem 3

In many cases one studies the field due to a static charge distribution $\rho(\vec{r}_0)$ which is unequal to zero only in a finite region (with a typical diameter $d$). at a point in space $\vec{r}$ far removed from that region, $d \ll r$.




a) Give the electrostatic potential.

b) Expand this potential to second order in $(d/r)$.

c) Give definitions of the total charge $q$, the dipole moment $\vec{p}$ and the quadrupole moment $\vec{Q}$ of the charge distribution.

d) What are the potentials due to a dipole $\vec{p}$ and to a quadrupole $\vec{Q}$.