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EKSAMEN I FAG 71516 ELEKTROMAGNETISK TEORI

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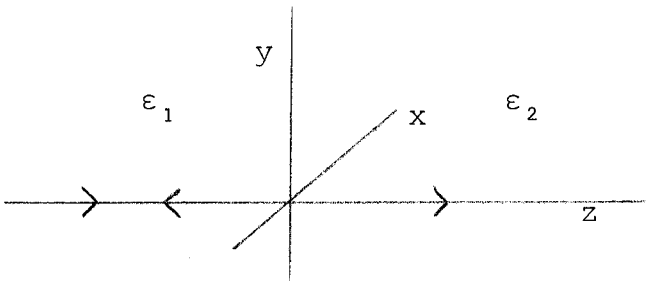
Tillatte hjelpemidler: Otto Øgrim: Størrelser og enheter i fysikken
K.Rottmann: Mathematische Formelsammlung
Regnestav/lommekalkulator

Problem 1

We consider the transmission and reflection of normally incident light. The boundary between the two media with dielectric constants ε_1 and ε_2 (while $\mu_1=\mu_2=1$ and $\sigma_1=\sigma_2=0$) is along the x-y plane. The refractive indices are defined by $n_1=\sqrt{\varepsilon_1}$ and $n_2=\sqrt{\varepsilon_2}$.

$$\vec{E}_i(\vec{r},t)=E_i(1,0,0)\exp\left[i\omega\left(\frac{n_1 z}{c}-t\right)\right]$$

$$\vec{\mathcal{H}}_i(\vec{r},t)=n_1 E_i(0,1,0)\exp\left[i\omega\left(\frac{n_1 z}{c}-t\right)\right]$$



the reflected light has the form

$$\vec{E}_r(\vec{r},t)=E_r(1,0,0)\exp\left[-i\omega\left(\frac{n_1 z}{c}+t\right)\right]$$

$$\vec{\mathcal{H}}_r(\vec{r},t)=-n_1 E_r(0,1,0)\exp\left[-i\omega\left(\frac{n_1 z}{c}+t\right)\right]$$

and the transmitted has the form

$$\vec{E}_t(\vec{r},t)=E_t(1,0,0)\exp\left[i\omega\left(\frac{n_2 z}{c}-t\right)\right]$$

$$\vec{\mathcal{H}}_t(\vec{r},t)=n_2 E_t(0,1,0)\exp\left[i\omega\left(\frac{n_2 z}{c}-t\right)\right]$$

a. Show that these fields satisfy Maxwells equations in a medium. (It is sufficient to do this for only one of the three fields).

b. Express E_r and E_t in terms of E_i using the boundary conditions.

Problem 2

a. Give the definition of the four potential A_μ in terms of the vector potential $\vec{A}$ and the normal potential ϕ .

b. What is the Lorentz gauge and explain why one has the freedom to choose the gauge.

c. Define the four current J_μ in terms of the current $\vec{J}$ and the charge density ρ .

d. Derive the (d'Alembert) equations for the four potential using the Lorentz gauge.

e. Give the definition of the field tensor $F_{\mu\nu}$ (a four by four tensor) and show how it is related to the electric and magnetic field.

f. Give the Maxwell equations for the field tensor.

g. Give the four dimensional Lorentz force in terms of $F_{\mu\nu}$ and J_μ . What is the physical meaning of the fourth component.

h. If the four potential A_μ is given in the rest frame give the four potential A'_μ in the frame of an observer moving with a velocity v along the x-axis.

Problem 3

An electron with charge e and mass m moves in a homogeneous magnetic field $\vec{B}=(0,0,B)$. The motion is orthogonal to the magnetic field in the x-y plane.

a. Show that the electron moves along a circle with a constant absolute velocity $V=|\vec{V}(t)|$. What is the rotation frequency. Give explicit expressions for the position $\vec{R}(t)$ and the velocity $\vec{V}(t)$ of the electron choosing the origin of the coordinate frame in the center of the circle.
(Use Newton's equation and not the relativistic equations of motion).

As the electron moves around a circle it will itself cause an electromagnetic field. The Liénard-Wiechert potentials due to the moving electron are given by

$$\phi(\vec{r},t)=\frac{e}{|\vec{r}-\vec{R}_r(t)|-\frac{1}{c}\vec{V}_r(t)\cdot(\vec{r}-\vec{R}_r(t))}$$

$$\vec{A}(\vec{r},t)=\frac{\frac{e}{c}\vec{V}_r(t)}{|\vec{r}-\vec{R}_r(t)|-\frac{1}{c}\vec{V}_r(t)\cdot(\vec{r}-\vec{R}_r(t))}$$

where the retarded position and velocity are given by

$$\vec{R}_r(t)=\vec{R}\left(t-\frac{|\vec{r}-\vec{R}_r(t)|}{c}\right) \quad \text{and} \quad \vec{V}_r(t)=\vec{V}\left(t-\frac{|\vec{r}-\vec{R}_r(t)|}{c}\right)$$

We shall now study the fields far away from the charge. Thus we assume $r\equiv|\vec{r}|$ to be large compared to the radius of the circle along which the electron moves.

b. Which approximation may be used in this case for $\vec{R}_r(t)$ and $\vec{V}_r(t)$.

c. Expand ϕ and $\vec{A}$ to linear order in the radius of the circle.

d. Calculate the resulting electric and magnetic field.

e. Give the contribution proportional to $1/r$ for long distances and argue that this is a radiation field.

f. Calculate the energy current emitted by the electron in the direction $\vec{r}/r$.

g. Calculate the average total energy loss of the electron due to radiation.

h. How does this effect the motion of the electron (no calculation, only give a qualitative description of the trajectory).