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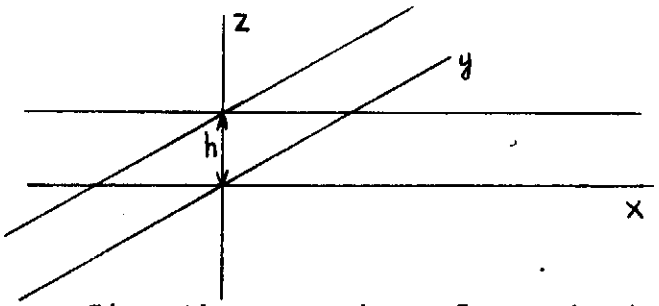
Tillatte hjelpemidler: K.Rottmann: Mathematische Formelsammlung

Feel free to answer the problems in Norwegian!
All symbols have the same meaning as in the lecture notes.

1. An incompressible fluid with density $\rho$ flows between two plane and parallel surfaces. The distance between the plates is $h$.




The flow is a consequence of a constant pressure gradient $\beta = \frac{\partial p}{\partial x}$ in the x direction.

a Give the equation of continuity and the Navier-Stokes equation for this problem.

b Solve them in order to obtain the velocity field $\vec{v}$ using stick boundary conditions on the $z=0$ and the $z=h$ surface.

c Calculate the total mass flow,per unit of length,between the surfaces.

d Give the Reynolds number for this flow in terms of the pressure gradient.

2. In an incompressible fluid the general solution of the equations for steady flow

$$\Delta\vec{v} = \frac{1}{\eta}\,\text{grad}\; p - \frac{1}{\nu}\vec{G} \quad \text{and} \quad \text{div}\;\vec{v} = 0\ ,$$

where $\vec{G}$ is the force density, is given by

$$\vec{v}(\vec{r}) = \frac{1}{8\pi\nu} \int d\vec{r}' \; \vec{G}(\vec{r}') \cdot \left[ \frac{1}{|\vec{r}-\vec{r}'|} \left( \vec{I} + \frac{(\vec{r}-\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2} \right) \right]$$

for the velocity field and

$$p(\vec{r}) = \frac{\rho}{4\pi} \int d\vec{r}' \; \vec{G}(\vec{r}') \cdot \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

for the pressure

a Give the socalled Stokeslet velocity and pressure fields due to a Stokeslet force density $\vec{G}(\vec{r}) = 8\pi\nu\vec{\alpha}\delta(\vec{r})$ .

b Verify that the Stokeslet fields indeed satisfy the above equations for $\vec{r} \neq 0$ .

c Give the force density for a stresslet, a rotlet and a potential doublet located in $\vec{r}=0$ .

d Calculate the resulting velocity fields for $\vec{r} \neq 0$ due to a stresslet, rotlet and a potential doublet located in $\vec{r}=0$ .

e Calculate the total force and the total torque exerted by a Stokeslet, a stresslet, a rotlet and a potential doublet on the fluid.

3. a A solid sphere is at rest with its center in $\vec{r}=0$ . Its radius is $R$ . Far from the sphere the velocity field is equal to $\vec{u}$ where $\vec{u}$ is independent of $\vec{r}$ and $t$ . Use stick boundary conditions on the surface. Construct the solution for the velocity field for all $\vec{r}$ using the velocity fields discussed in the previous problem (i.e. the Stokeslet, stresslet, rotlet and potential doublet fields). Calculate the force and torque exerted on the sphere by the fluid.

b Do the same as in 3a but now for the case that the velocity far from the sphere is given by

$$\vec{u}(\vec{r}) = \vec{\Omega} \times \vec{r}$$

In this case $\vec{\Omega}$ is independent of $\vec{r}$ and $t$ . Do not forget to calculate the force and the torque exerted on the sphere by the fluid.