UNIVERSITETET I TRONDHEIM
NORGES TEKNISKE HØGSKOLE
INSTITUTT FOR TEORETISK FYSIKK

Faglig kontakt under eksamen:
F.Bakke/L.S.Schulman
Tlf. 3649

EKSAMEN I FAG 71560 TEORETISK FYSIKK
VIDEREGÅENDE KURS B

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kl.0900-1500

Tillatte hjelpemidler: Kalkulator, håndskrevne notater (ikke trykt stoff)

Kandidatene velger enten oppgavesett A eller oppgavesett B.

Kandidatene kan besvare oppgavene på norsk eller engelsk.

Oppgavesett A

Problem 1

Consider site percolation in one dimension, each site having probability p of being occupied, $0 \leq p \leq 1$. In fact this is not a very interesting problem for reasons that you will soon see, but it is solvable and standard techniques work without difficulty. The sites are labeled $k=0,\pm 1,\pm 2, \ldots$.

(a) First apply renormalization group techniques in the following way. Combine each even numbered site (call it $2k$) with its neighbor to the right ($2k+1$) to form a block. These blocks will be the new sites after the rescaling. We require a rule for deciding whether or not the new sites are occupied, for example one may require both constituent old sites to be occupied in order for the new site to be considered occupied or it may be enough for only one of the two to be occupied.

(a 1) Find the correct rule and justify its appropriateness.

(a 2) With this rule and for some value of p for the old sites, find the occupation probability p' for the new sites.

(a 3) In (a 2) you should have found a function $p'=f(p)$. Find the fixed points of this transformation and identify them as attractors or as repellors (i.e., when p is near a particular fixed point, is $f(p)$ closer to or farther from the fixed point.)

(a 4) Based on (a 3) what is the critical p_c for this problem ?

(b) Let $g(k)$ be the probability that two sites a distance k apart are part of the same cluster. Define a correlation length $\xi(p)$ by $g(k) \sim \exp(-k/\xi)$. (In general this relation need be only asymptotic in k , although there is no reason for it not to be correct for all k).

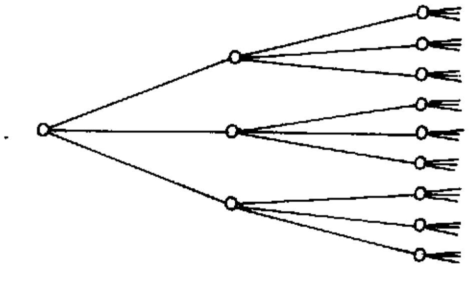
(b 1) Find $\xi(p)$.

(b 2) Using the p_c of (a 4) note that $\xi(p) \to \infty$ as $p \to p_c$. (If you don't get this, either your p_c is wrong or your $\xi(p)$ is wrong). As usual the exponent ν is defined by $\xi(p) \sim (p_c-p)^{-\nu}$. Find ν .

(c) In fact, if all one wishes is ν , the explicit form of ξ found in (b 1) is unnecessary.

(c 1) Find ν using only the information in (a 3), (a 4) and your knowledge of the behavior of ξ under the scale transformation of part (a).

Problem 2

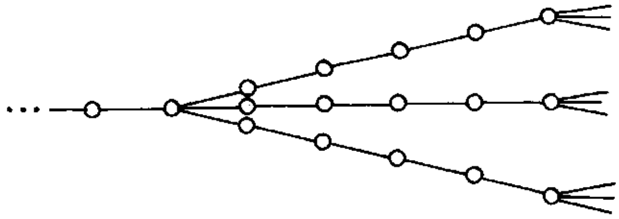


A Bethe lattice of coordination number 4 is illustrated above; that is, 3 potential bonds emanate to the right of each site.

(a) Using results from self-avoiding walks, find a bound for the critical probability p_c for bond percolation.

(b) Find the exact value of p_c for bond percolation.

(c) Suppose each line from one vertex to the next consists of 5 bonds and 4 sites as illustrated below.



If each bond is occupied with probability p , what is now the critical value for bond percolation on this lattice ? (Hint: You may find part b of problem 1 on this examination useful.)

Problem 3

Consider the site percolation problem for the two-dimensional square lattice. Call some particular site the "origin". As discussed in the lectures the probability P_s that the origin is part of a cluster of precisely s sites is given by a sum $p^s \sum_t a_{st} q^t$ with p the probability of site occupation, $q=1-p$, a_{st} the number of marked clusters with perimeter t . The perimeter is the number of vacant sites needed to isolate a cluster and by "marked" is meant that different locations of the origin within a cluster are counted separately. In what follows we will count unmarked clusters, in which no special site is selected and call the number of (unmarked) clusters with s sites and perimeter t g_{st} . The relation is $a_{st}=sg_{st}$. The expression $D_s=\sum_t g_{st} q^t$ is called the perimeter polynomial and for the square lattice (in 2 dimensions) $D_1=q^4$, $D_2=2q^6$ and $D_3=4q^7+2q^8$.

(a) Find D_4 (showing how you count diagrams to obtain your result).

(b) The expression for D_{10} is

$4q^{11}+ 154q^{12}+ 986q^{13}+ 3676q^{14}+ 7612q^{15}+ 9750q^{16}$

$+ 8192q^{17}+ 4330q^{18}+ 1416q^{19}+ 292q^{20}+ 32q^{21}+ 2q^{22}$.

Describe the diagrams used to obtain the last two terms (q^{21} and q^{22}) and give the coefficient of q^{25} in D_{12} .

(c) For the site problem with $p<p_c$ you know that $p=\sum_{s=1}^{\infty} P_s$, which in this case implies $p=\sum s p^s D_s$.

(N.B. D_s is a function of p because of $q=1-p$).

(c 1) If the sum is cut off at $s=4$, what order in p is the error, i.e. what is the lowest power of p in

$$p-\sum_{s=1}^{4} sp^s D_s \ ?$$

(c 2) Use your result in part (c 1) to deduce the total number of 5-site unmarked diagrams (which equals D_5 with q set equal to one).

(d) Recall that the expected cluster size (for $p<p_c$) is

$S(p)=\frac{1}{p}\sum_{s=1}^{\infty} s^2p^sD_s$. Evaluate enough terms to get all powers of p up to and including p^3 . Estimate p_c using the [2,1] Padé approximant, i.e.setting $S(p)$ equal $(A+Bp+Cp^2)/(1+Dp)$.

How good do you consider this estimate (give reasons) ?

Problem 4

In the lectures it was recalled that for Brownian motion $(\Delta x)^2 \sim \Delta t$ and that this feature implies that the (continuous) path $\vec{x}(t)$ of a particle undergoing Brownian motion in $\mathbb{R}^d$ is a fractal subset of $\mathbb{R}^d$ $(d\geq 2)$ with fractal dimension 2. The self-avoiding walk (SAW) is found to depart from the origin rather more rapidly (using relative probabilities given by Brownian motion, but discarding self-interacting paths), and for $\mathbb{R}^2$ the best contemporary information suggests $(\Delta x)^{\frac{4}{3}} \sim \Delta t$. When viewed on an appropriately gross scale (so that any underlying lattice is invisible) the SAW trajectory will be fractal. What is its fractal dimension ?

Oppgavesett B

Oppgave 1

Et fritt vektor-meson-felt $W^{\mu}(x)$ beskrives ved Lagrange-tettheten

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} \kappa^2 W_{\mu} W^{\mu} \qquad (\kappa = \text{konstant})$$

hvor

$$G_{\mu\nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} \quad .$$

a) Finn feltlikningen og vis at feltet må oppfylle Lorentzbetingelsen $\partial_{\nu} W^{\nu} = 0$.

b) Utled en energibevarelseslikning for feltet og skriv opp den kanoniske energi-impulstensoren.

c) Redegjør for hvordan en kan symmetrisere den kanoniske energi-impulstensoren og vis at en her da kan få en symmetrisk energi-impulstensor av formen

$$T^{\mu\nu} = -G^{\mu}{}_{\lambda} G^{\nu\lambda} + \kappa^2 W^{\mu} W^{\nu} - \delta^{\mu\nu} \mathcal{L}$$

d) En plan monokromatisk sirkulærpolarisert mesonbølge har formen

$$\vec{W} = a\sqrt{\frac{\hbar}{\omega}}\, c \,\left(\hat{e}_x \sin(kz - \omega t) + \hat{e}_y \cos(kz - \omega t)\right) \quad ,$$

Finn bølgens energi- og impuls-tetthet uttrykt ved amplitudefaktoren a og angi hvilken masse disse verdiene antyder for dette vektor-mesonet.

Oppgave 2

a) Et koordinatsystem K (koordinater x^{μ}) som ved $t=0$ ligger i ro i og har felles origo med et inertialsystem K_0 (koordinater X^{μ}), utsettes for jevn akselerasjon a i forhold til K_0. Angi sammenhengen mellom koordinatene i de to systemene for en hendelse og finn komponentene for den metriske tensor i K (Anta $t<\frac{c}{a}$)

b) I et konstant homogent gravitasjonsfelt er intervallet ds gitt ved

$$ds^2 = \left(1 - \frac{g^2t^2}{c^2}\right) c^2dt^2 + 2gt\, dtdx - dx^2 - dy^2 - dz^2 \quad , \; (g=\text{konstant}) \; .$$

Finn ut fra virkningsintegralet $S=-mc\int ds$ bevegelseslikningene

$$u\left(\frac{dx}{ds}, \frac{dt}{ds}, t\right) = A \quad , \quad v\left(\frac{dy}{ds}\right) = B \quad , \quad w = \left(\frac{dz}{ds}\right) = C \quad , \quad (A,B,C=\text{konstant})$$

for en partikkel i dette gravitasjonsfeltet og vis at partikkelens bane vil ligge i et plan.

c) Benytt likningen $g_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} = 1$ til å finne bevegelseslikningen

$\frac{dx}{dt} = f(t)$ for partikkelen i gravitasjonsfeltet og løs den.

Sammlikn med forholdene i a) .