

OPPG1

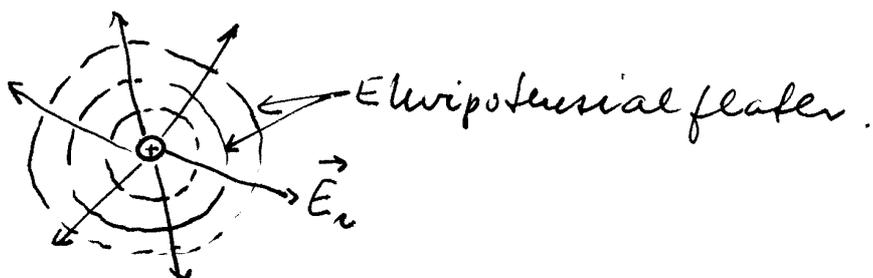
a) Det elektriske potensialet fra ei punkt-ladning i avstand r fra denne:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

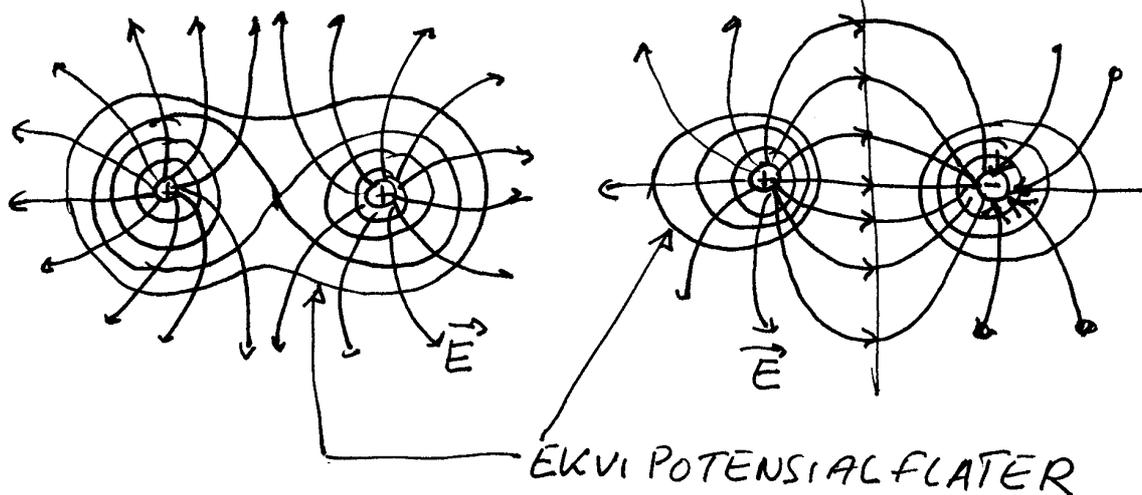
$$E_r = -\frac{dV}{dr} = -\frac{q}{4\pi\epsilon_0} \frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Symmetrien tilsier bare radiale felt, dvs:

$$\vec{E}_r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r}$$



b) Ekvipotensialflater: flater $\perp \vec{E}$



c) V kan vi skrive ned ved ved \vec{a} observere avstandene til P i figuren:

$$V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(x^2 + (y-a)^2 + z^2)^{\frac{1}{2}}} - \frac{q}{(x^2 + (y+a)^2 + z^2)^{\frac{1}{2}}} \right]$$

$$\vec{E} = -\vec{\nabla} V ; \text{ eller } E_x = -\frac{\partial V}{\partial x} ; E_y = -\frac{\partial V}{\partial y} ; E_z = -\frac{\partial V}{\partial z}$$

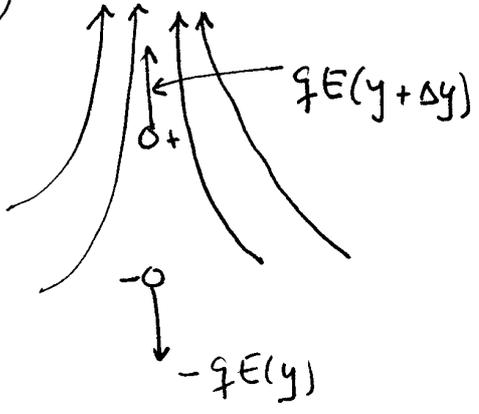
Dus:

$$\underline{E_x} = -\frac{\partial V}{\partial x} = \frac{q}{4\pi\epsilon_0} \left\{ \frac{x}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\}$$

$$\underline{E_y} = -\frac{\partial V}{\partial y} = \frac{q}{4\pi\epsilon_0} \left\{ \frac{y-a}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\}$$

$$\underline{E_z} = -\frac{\partial V}{\partial z} = \frac{q}{4\pi\epsilon_0} \left\{ \frac{z}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{z}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\}$$

a)



Total kraft på dipolen

$$F = qE(y + \Delta y) - qE(y)$$

Med liten Δy kan vi skrive:

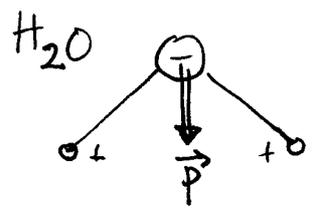
$$E(y + \Delta y) \approx E(y) + \frac{dE}{dy} \Delta y$$

$$F = q \left[E(y) + \frac{dE}{dy} \Delta y - E(y) \right]$$

$$\underline{F} = q \Delta y \cdot \frac{dE}{dy} = \underline{p \frac{dE}{dy}}$$

$$E = E(y) = C \cdot y \Rightarrow \underline{F = pC}$$

I homogent felt: $E = \text{konstant} \Rightarrow \underline{F = p \frac{dE}{dy} = 0}$

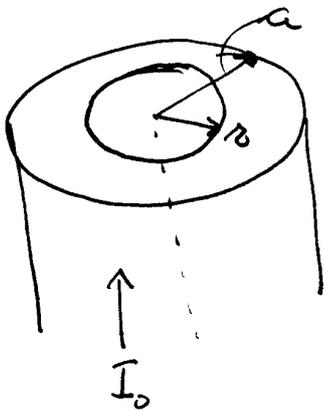
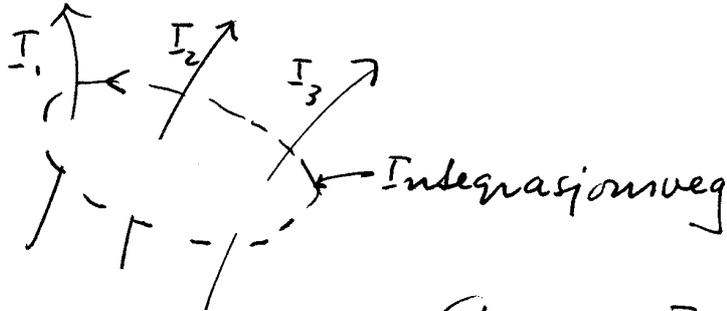


H₂O har ein kraftig elektrisk dipol \vec{p} . Denne blir ikkje påverka av ei nettokraft for translasjon, men blir vridd av det elektromagnetiske feltet

p.g.a at vridemomentet er $\vec{\tau} = \vec{p} \times \vec{E}$. Rotasjonsrørsle i E-feltet medfører energi-overføring frå E-feltet til omsivelsane til H₂O p.g.a dette.

OPPG 2 :

a) Ampères lov $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ seier at \vec{B} -feltet integrert langs ei lukka kurve er like μ_0 ganger netto straum som kurva omsluttar. Utsegnet er illustrert i figuren.



Strøm I innefor ein radius $r \leq a$:

$$I = \frac{\pi r^2}{\pi a^2} I_0 = \frac{r^2}{a^2} I_0$$

Ampère:

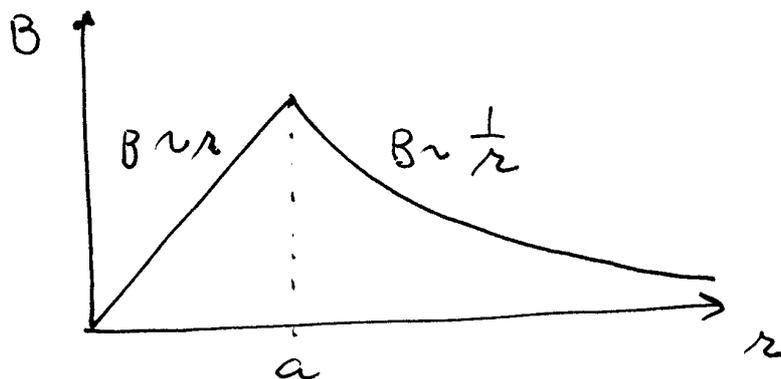
$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \cdot 2\pi r$$

da $\vec{B} \parallel d\vec{l}$.

$r \leq a$: $B \cdot 2\pi r = \mu_0 \frac{r^2}{a^2} I_0$

$$\underline{\underline{B = \frac{\mu_0 I_0}{2\pi} \cdot \frac{r}{a^2}}}$$

$r \geq a$: $B \cdot 2\pi r = \mu_0 I \Rightarrow \underline{\underline{B = \frac{\mu_0 I_0}{2\pi r}}}$





$r \leq a_1 : I(r) = 0$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot 0$

$\vec{B} = 0$

$a_1 \leq r \leq a_2$: $I(r)$: Strom innerhalb radius r

$\frac{I(r)}{I_0} = \frac{\pi r^2 - \pi a_1^2}{\pi a_2^2 - \pi a_1^2} = \frac{r^2 - a_1^2}{a_2^2 - a_1^2} = \frac{(2r^2 - a_1^2)}{a_2^2 - a_1^2}$

$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 I(r) = \frac{(2r^2 - a_1^2)}{a_2^2 - a_1^2} \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r} \frac{(2r^2 - a_1^2)}{a_2^2 - a_1^2} = \frac{\mu_0 I_0}{2\pi (a_2^2 - a_1^2)} \cdot \frac{(2r^2 - a_1^2)}{r}$

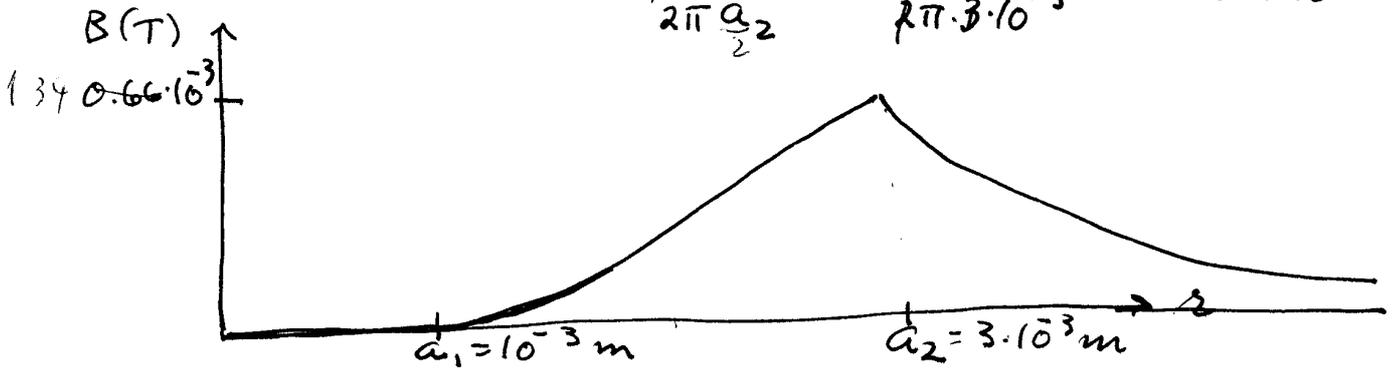
$a_2 \leq r$: $\oint \vec{B} \cdot d\vec{l} = 2\pi r \cdot B = \mu_0 I_0$

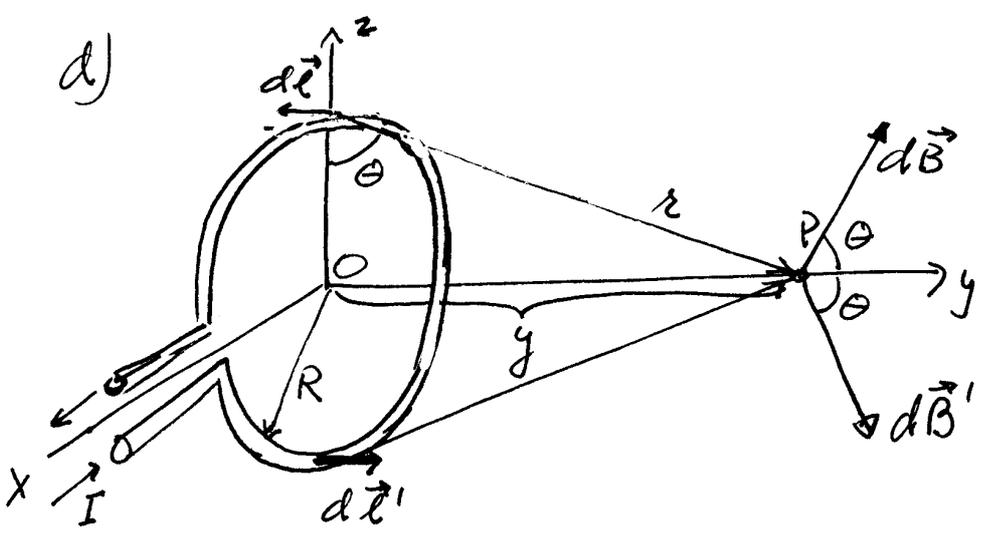
$B = \frac{\mu_0 I_0}{2\pi r}$

c) $I_0 = 10A$; $a_1 = 1mm = 10^{-3}m$; $a_2 = 3mm = 3 \cdot 10^{-3}m$

Near $2r = a_1$: $B \approx \frac{\mu_0 I_0 \cdot 2}{2\pi (a_2^2 - a_1^2)} \cdot \frac{2r - a_1}{r} = \frac{\mu_0 I_0}{\pi (a_2^2 - a_1^2)} \left[2 - \frac{a_1}{r} \right]$

Ved $2r = a_2$: $B = \frac{\mu_0 I_0}{2\pi \frac{a_2}{2}} = \frac{4\pi \cdot 10^{-7} \cdot 10}{\pi \cdot 3 \cdot 10^{-3}} T = \frac{1.34}{0.67} \cdot 10^{-3} T$





P.g. a symmetri haussellerer komponentane av $d\vec{B}$ og $d\vec{B}'$ normalt til y -aksen. \vec{B} må derfor bli en vektor langs y .

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{y^2 + R^2}$$

$$\vec{B} = \hat{j} \oint dB \cos \theta = \hat{j} \frac{\mu_0 I}{4\pi} \int \frac{\cos \theta dl}{y^2 + R^2}$$

$$\cos \theta = \frac{R}{(y^2 + R^2)^{1/2}}$$

$$\vec{B} = \hat{j} \frac{\mu_0 I R}{4\pi (y^2 + R^2)^{3/2}} \oint dl = \frac{\mu_0 I R^2}{2(y^2 + R^2)^{3/2}} \hat{j}$$

I origo, 0 : $y = 0$

$$\vec{B} = \frac{\mu_0 I}{2R} \hat{j}$$

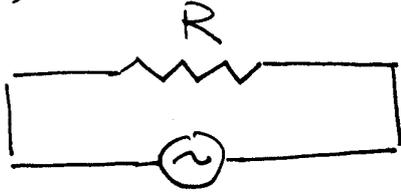
$R = 1 \text{ cm} = 10^{-2} \text{ m}$; $I = 10 \text{ A}$:

$$\vec{B} = \frac{4\pi \cdot 10^{-7} \text{ H/m} \cdot 10 \text{ A}}{2 \cdot 10^{-2} \text{ m}} = 0.63 \cdot 10^{-3} \text{ T} = \underline{\underline{0.63 \text{ mT}}}$$

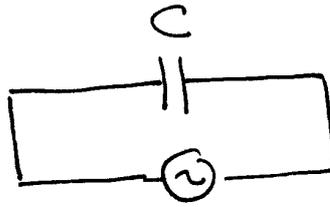
OPP63

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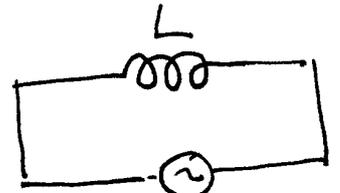
a)



i)



ii)



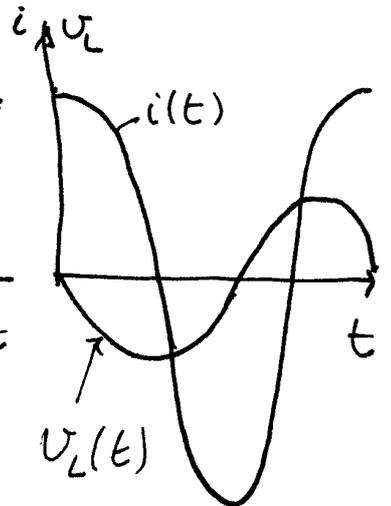
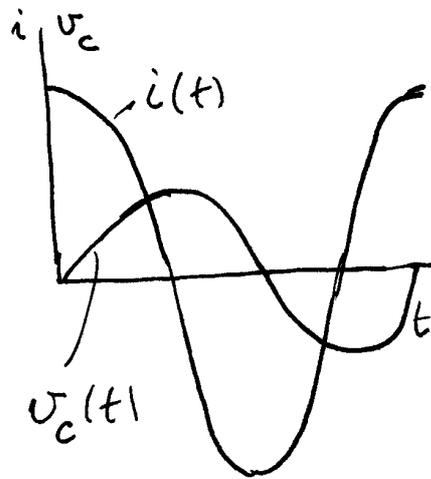
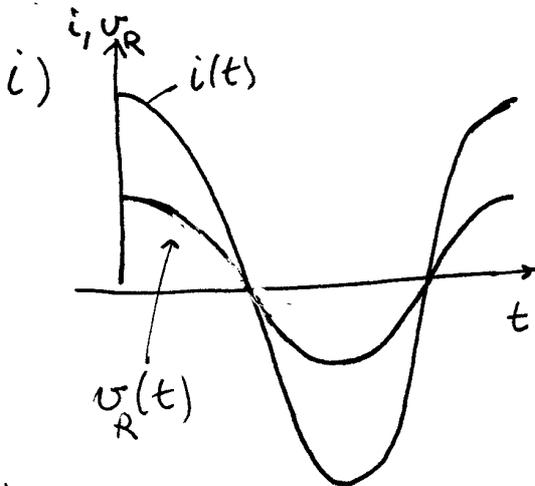
iii)

Für alle: $i(t) = I \cos \omega t$.

i) $\underline{U_R} = i R = I R \cos \omega t = \underline{V_R \cos \omega t}$; $V_R = I R$

ii) $\underline{U_C} = \frac{q}{C} = \frac{1}{C} \int_0^t i dt = \frac{I}{C} \int_0^t \cos \omega t dt = \underline{\underline{\frac{I}{\omega C} \sin \omega t}}$

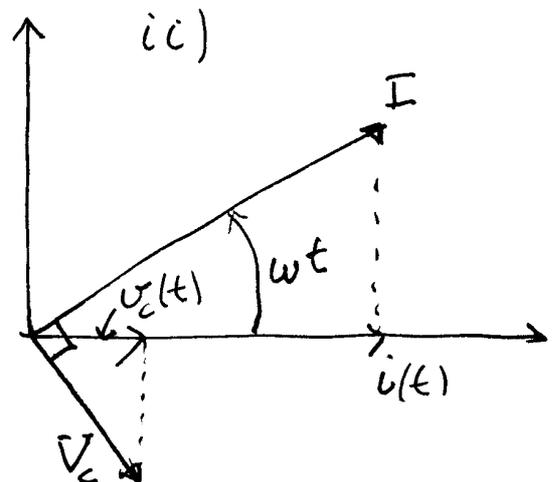
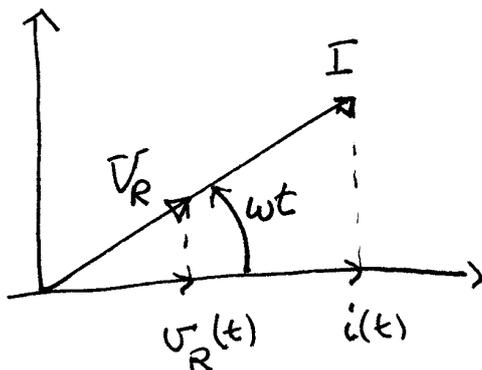
iii) $\underline{U_L} = L \frac{di}{dt} = L \frac{d}{dt} (I \cos \omega t) = \underline{\underline{-I \omega L \sin \omega t = I \omega L \cos(\omega t + \frac{\pi}{2})}}$

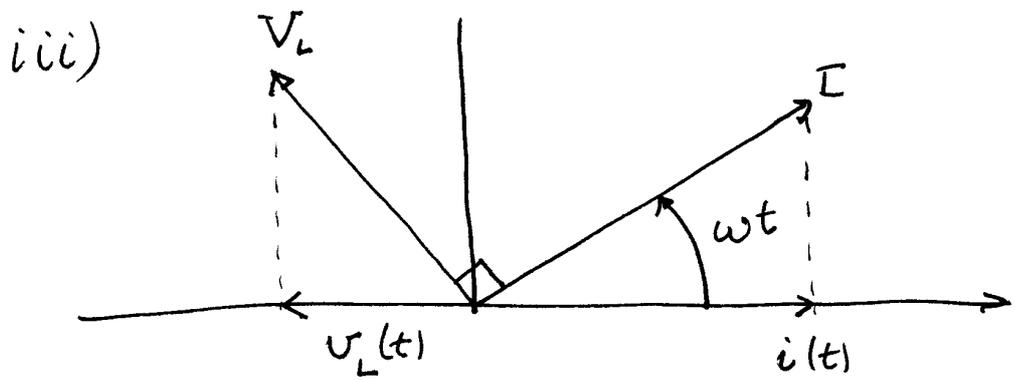


b)

Phasordiagramm

i)





Fråustillingane i a) og b) svararar fordi projeksjonane av I og V ned på horisontalakse svararar med uttrykket i a), og fasevinklane er h.h.v. $0, -90^\circ$ og $+90^\circ$ både i dei matematiske uttrykket og i figurane.

c) Reaktansene finn vi av svara i a)

$$V_c = V_c \cos(\omega t - \frac{\pi}{2}) = \frac{I}{\omega C} \cos(\omega t - \frac{\pi}{2})$$

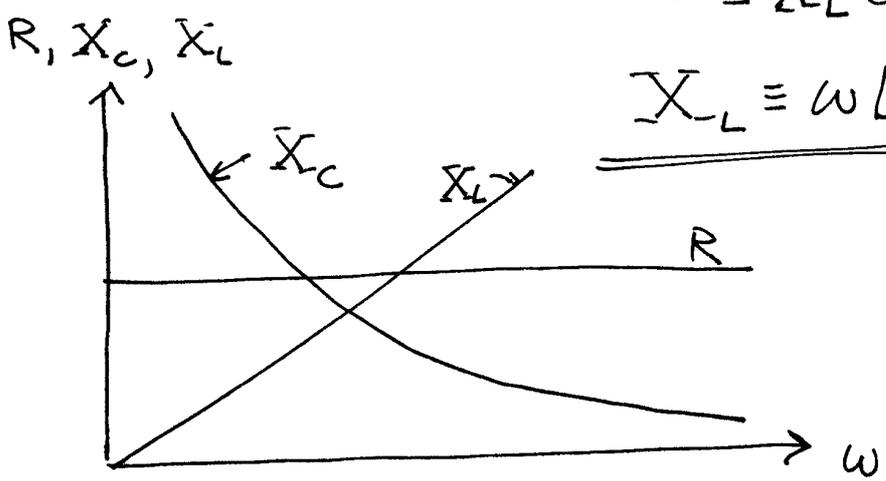
$$\equiv I X_c \cos(\omega t - \frac{\pi}{2})$$

$$\underline{X_c \equiv \frac{1}{\omega C}}$$

$$V_L = V_L \cos(\omega t + \frac{\pi}{2}) = I \omega L \cos(\omega t + \frac{\pi}{2})$$

$$\equiv I X_L \cos(\omega t + \frac{\pi}{2})$$

$$\underline{X_L \equiv \omega L}$$



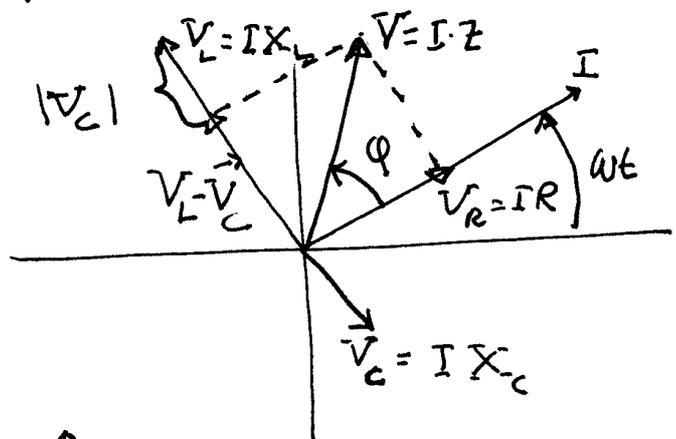
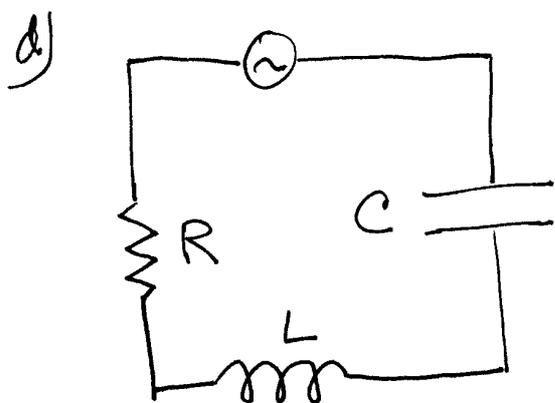
Kapazitiv reaktans $X_C \rightarrow 0$ när $\omega \rightarrow \infty$

Induktiv reaktans $X_L \rightarrow \infty$ när $\omega \rightarrow 0$

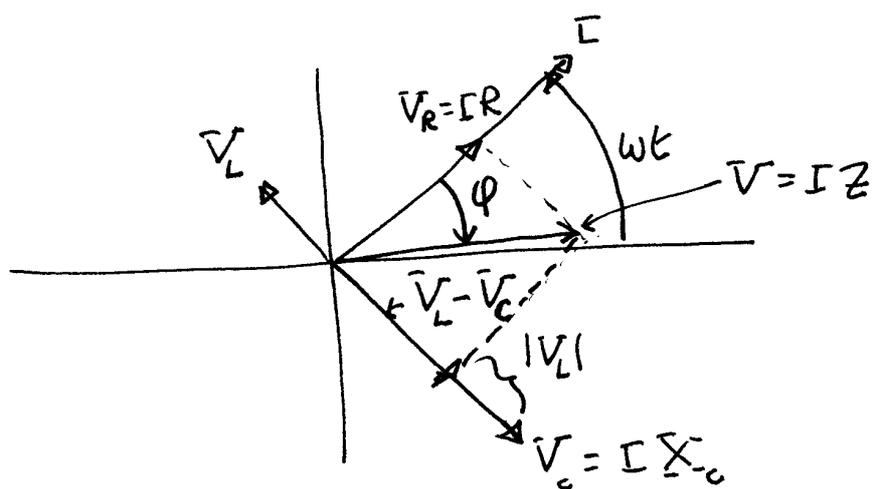
Kapazitiv reaktans $X_C \rightarrow \infty$ när $\omega \rightarrow 0$

Induktiv reaktans $X_L \rightarrow 0$ när $\omega \rightarrow \infty$

Konsekvensen av detta är att en kondensator slipper genom ac-ström, men blockerar dc-ström; men för en spole är det omvänt.



VISARDIAGRAM FÖR $X_L > X_C$



VISARDIAGRAM FÖR $X_L < X_C$

Av Fig ser vi att

$$\underline{\underline{\tan \phi = \frac{V_L - V_C}{V_R} = \frac{I\omega L - I\frac{1}{\omega C}}{IR} = \frac{\omega L - \frac{1}{\omega C}}{R}}}$$

OPPG 4.

$$a) \quad \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2} A \sin(\omega t + kx) = -Ak^2 \sin(\omega t + kx)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2} A \sin(\omega t + kx) = -A\omega^2 \sin(\omega t + kx)$$

$$): \frac{1}{k^2} \frac{\partial^2 y}{\partial x^2} = \frac{1}{\omega^2} \frac{\partial^2 y}{\partial t^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{(\omega/k)^2} \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

1): $A \sin(\omega t + kx)$ oppfyller b. likn. med $v = \omega/k$

Bølgefart v (fasefart) kan vi definere som farten til et punkt med fast fase.

For $A \sin(\omega t + kx)$: $\omega t + kx = \text{constant}$

$$): \quad \frac{d}{dt}(\omega t + kx) = 0$$

$$\frac{dx}{dt} = -\frac{\omega}{k} = -v$$

Fasefarten er negativ 1): Bølge langs $-x$

$$b) \quad y = 4.00 \text{ cm} \sin\left(2\pi \frac{t}{0.030 \text{ s}} - \frac{x}{50.0 \text{ cm}}\right)$$

$$\equiv A \sin(\omega t - kx) = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Amplitude: $A = 4.00 \text{ cm}$;

Frekvens: $\omega = \frac{2\pi}{0.030 \text{ s}} = 2\pi f \Rightarrow f = \underline{\underline{33.33 \text{ Hz}}}$

Bølglengde: $\lambda = 50 \text{ cm} = 0.5 \text{ m}$

Fart: $v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f \cdot \lambda = 33.33 \text{ Hz} \cdot 0.5 \text{ m} = \underline{\underline{16.7 \text{ m/s}}}$

$$c) \quad \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2} = \frac{1}{(F/\mu)} \frac{\partial^2 y}{\partial t^2}$$

$$\therefore v = (F/\mu)^{1/2}$$

$$\underline{v} = (80 \text{ N} / (0.2 \text{ kg} / 10 \text{ m}))^{1/2} = \underline{63.2 \text{ m/s}}$$

$$\underline{\lambda} = v/f = \frac{63.2 \text{ m/s}}{6 \text{ s}^{-1}} = \underline{10.5 \text{ m}}$$

$$\lambda = \frac{(F/\mu)^{1/2}}{f} \Rightarrow f = \frac{(F/\mu)^{1/2}}{\lambda} = \frac{(160 \text{ N} \cdot 10 \text{ m} / 0.2 \text{ kg})^{1/2}}{10.5 \text{ m}}$$

$$\underline{f = 8.5 \text{ Hz}}$$

d)

$$y(x,t) = y_1 + y_2 = A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$

Ved bruk av $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$:

$$y(x,t) = 2A \sin kx \cdot \cos \omega t$$

Dvs: Alle utsving er i fase.

Dette kan vi ta som definisjon av stående bølge.

Posisjon av knutar bestemmes av

$$\sin kx = 0$$

$$kx = 0, \pi, 2\pi, 3\pi, \dots$$

$$x = 0, \frac{\pi}{k}; \frac{2\pi}{k}; \frac{3\pi}{k}; \dots$$

$$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \dots$$

stille spørsmål:

Shelket i
 strengen bestemmes
 v og dermed
 bølglengden λ
 og fullevisene f_n

Normalmoder med feste ender: $\lambda_n = \frac{2L}{n}$; $n = 1, 2, 3, \dots$
 $f_n = v/\lambda_n = n(v/2L) = n f_1$