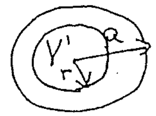
LØSNINGSFORSLAG fra maskin. høst. – 93 (eksam

Oppgave 1.

a) Gauss lov: $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ (i vakuum)

Ladning pr. volumenhet: $\rho = \frac{Q}{\frac{4}{3}\pi a^3}$

$r < a$ $\oint \vec{E} \cdot d\vec{A} = \frac{\sum q \text{ innenfor}}{\epsilon}$



$\sum q_{\text{innenfor}} = \rho \cdot V'$ hvor $V' = \frac{4\pi r^3}{3}$

$\sum q_{\text{innenf.}} = \rho \cdot \frac{4\pi r^3}{3} = \frac{Q \cdot \frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = \frac{Q \cdot r^3}{a^3}$

$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{Q \cdot r^3}{\epsilon \cdot a^3}$

$\vec{E} \perp d\vec{A}$ over hele Gaussflaten $\Rightarrow$

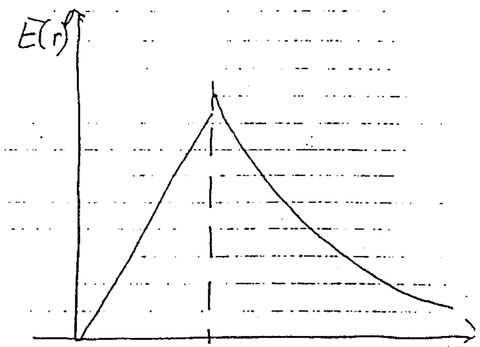
$E(r) \cdot A = \frac{Q \cdot r^3}{\epsilon \cdot a^3} \Leftrightarrow E(r) \cdot 4\pi r^2 = \frac{Q \cdot r^3}{\epsilon R^3}$

$\Leftrightarrow E(r) = \frac{Q \cdot r}{4\pi\epsilon a^3}$ $\quad r \xrightarrow{<} a \Rightarrow E(r) = \frac{Q}{4\pi\epsilon a^2}$

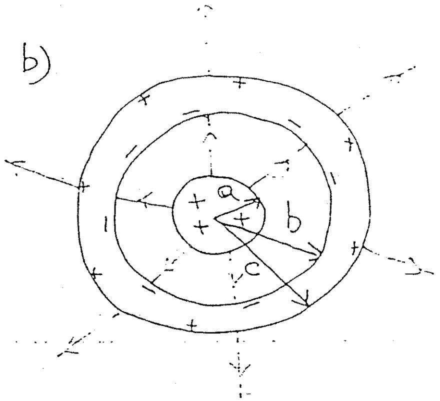
$r > a$ $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \Leftrightarrow E(r) = \frac{Q}{\epsilon_0 \cdot 4\pi r^2}$

$r \xrightarrow{>} a \Rightarrow E(r) \Rightarrow \frac{Q}{4\pi\epsilon_0 a^2}$

$\epsilon > \epsilon_0$



b)



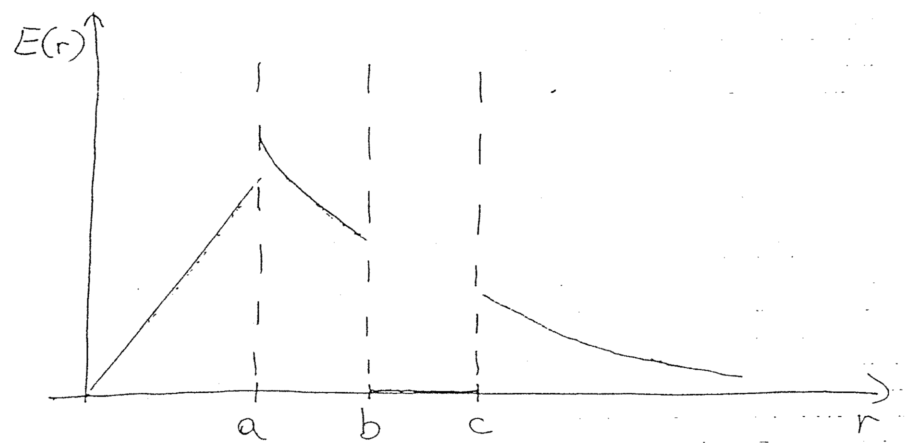
E = 0 inni metallet.
⇒ $\vec{E} \perp$ metalloverflaten.
(ingen bevegelse av ladninger)

bet. $r < a$ ⇒ E som tidligere: $E = \frac{Q \cdot r}{4\pi\epsilon a^3}$

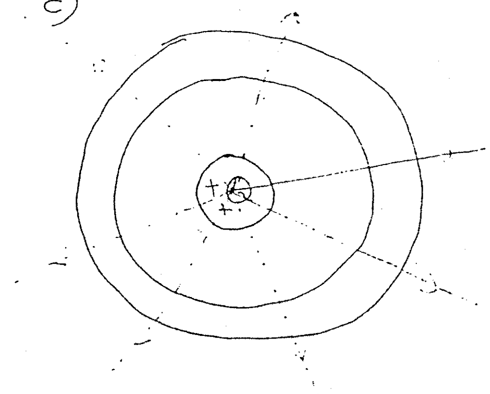
bet. $a < r < b$ ⇒ $E = \frac{Q}{4\pi\epsilon_0 r^2}$

$b < r < c$ ⇒ $E = 0$ inni lederen.

$r > c$ ⇒ $E = \frac{Q}{4\pi\epsilon_0 r^2}$



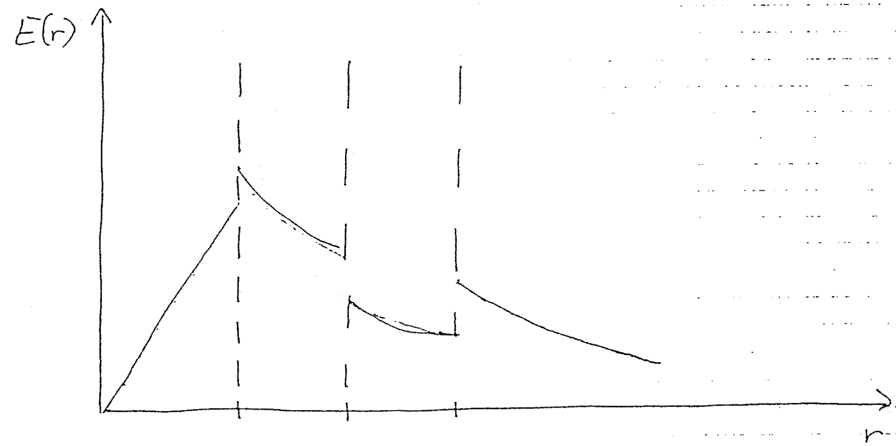
c)



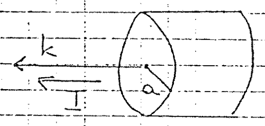
$r < a$ som i pkt b.
$a < r < b$ — " —

$b < r < c$ $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon} \Leftrightarrow E = \frac{Q}{4\pi\epsilon r^2}$

$r > c$ som i pkt b.



Oppgave 2



a) for $r \le a$

$$\vec{j} = \frac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]\vec{k}$$

$$j = \frac{dI}{dA}$$

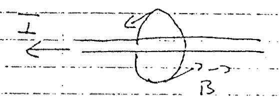
$$dI = j\,dA = j\cdot 2\pi r\,dr$$

$$\Rightarrow I = \int_0^a \frac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]2\pi r\,dr$$

$$\Leftrightarrow I = \frac{4I_0}{a^2}\int_0^a \left\{r - \frac{r^3}{a^2}\right\}dr = \frac{4I_0}{a^2}\left[\frac{r^2}{2} - \frac{r^4}{4a^2}\right]_0^a$$

$$\Leftrightarrow \underline{\underline{I}} = \frac{4I_0}{a^2}\left\{\frac{a^2}{2} - \frac{a^2}{4}\right\} = \underline{\underline{I_0}}$$

b) Ampères lov: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{innenfor}$



$\vec{B}$ tangent til sirkel
$|\vec{B}|$ konst. i samme avstand fra leder. Velger sirkel som integrasjonssløyfe

$\Rightarrow \vec{B} \parallel d\vec{l}$

$$\Rightarrow B\cdot 2\pi r = \mu_0 I_0 \Leftrightarrow \underline{\underline{B = \frac{\mu_0 I_0}{2\pi r} \quad \text{for } r \ge a}}$$

c) for $r < a$,

$$\underline{\underline{I(r)}} = \int_0^r j(r)\,dA = \int_0^r \frac{2I_0}{\pi a^2}\left\{1-(r/a)^2\right\}2\pi r\,dr$$

$$= \frac{4I_0}{a^2}\left\{\frac{r^2}{r} - \frac{r^4}{4a^2}\right\} = \underline{\underline{\frac{I_0 r^2}{a^2}\left\{2 - \frac{r^2}{a^2}\right\}}}$$

d) $B(r)$ har samme symmetri som i b.

Ampères lov: $\oint_{sirkel} \vec{B}\cdot d\vec{l} = \mu_0 I_{innen\ for}$

$$\Leftrightarrow \vec{B} \parallel d\vec{l} \Rightarrow B\cdot 2\pi r = \mu_0 \frac{I_0 r^2}{a^2}\left[2 - \frac{r^2}{a^2}\right]$$

$$\Leftrightarrow \underline{\underline{B = \frac{\mu_0 I_0 r}{2\pi a^2}\left[2 - \frac{r^2}{a^2}\right]}}$$

fra b: $\underline{r = a} \Rightarrow \underline{\underline{B(a) = \frac{\mu_0 I}{2\pi a}}}$

fra d: $r = a \Rightarrow \underline{\underline{B(a)}} = \frac{\mu_0 I a}{2\pi a^2}\left[2 - \frac{a^2}{a^2}\right] = \underline{\underline{\frac{\mu_0 I}{2\pi a}}}$

som ventet!

e) koaksialkabel.

Magnetfelt fra indre leder for $r > R$: $B(r) = \frac{\mu_0}{2\pi}$

Fra ytre leder:

$$\oint_{sirkel} \vec{B}_y\cdot d\vec{l} = \mu_0\cdot I \Leftrightarrow B_y\cdot 2\pi r = \mu_0\cdot I$$

$$B_y = -B(r) \Rightarrow -\frac{\mu_0 I_0}{2\pi r}\cdot 2\pi r = \mu_0\cdot I$$

$\Leftrightarrow \underline{I = -I_0}$ ~~xxxx~~ motsatt rettet men like store

Oppgave 3.

a) Konstruktiv interferens for:

$d \sin\theta = m\lambda$; $m = 0, \pm 1, \pm 2, \dots$

2. ordens hovedmaksimum: $m = \pm 2$.

$\lambda_a = 589{,}0\,nm$: $\theta_a = \pm \arcsin\left(\frac{2\lambda_a}{d}\right)$

$\Leftrightarrow \underline{\theta_a} = \pm \arcsin\left(\frac{2 \cdot 589{,}0 \cdot 10^{-9} m}{2 \cdot 500 \cdot 10^{-6} m}\right) = \underline{\underline{\pm 28{,}11}}$

$\lambda_b = 589{,}6\,nm$: $\theta_b = \pm \arcsin\left(\frac{2\lambda_b}{d}\right)$

$\underline{\theta_b} = \pm \arcsin\left(\frac{2 \cdot 589{,}6 \cdot 10^{-9} m}{2 \cdot 500 \cdot 10^{-6} m}\right) = \underline{\underline{\pm 28{,}14^\circ}}$

b) Antall hovedmax er begrenset av at:

$|\sin\theta| \leq 1 \Rightarrow \frac{|m|\lambda_a}{d} \leq 1$

$\Rightarrow |m| \leq \frac{d}{\lambda_a} = \frac{2 \cdot 500 \cdot 10^{-6}}{589{,}0 \cdot 10^{-9}} = \underline{4{,}244}$

$\Rightarrow |m| \leq 4 \Rightarrow m = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Det er mulig å observere <u>9 hovedmax</u> for λ_a.

c) $I = I_{diffr.} \cdot I_{interf.}$

Utslukning i diffraksjonsleddet vil gi 0 intensitet i 3. ordens hovedmax for interferens.

<u>utslukning:</u>

$\sin\theta_{diff} = \frac{m_{diff} \cdot \lambda}{D}$ $\quad m_{diff} = \pm 1, \pm 2 \dots$

$\sin\theta_{diff} = \frac{m_{diff} \cdot \lambda_a}{D} = \sin\theta_a = \frac{3\lambda_a}{d}$

$\Leftrightarrow \frac{m_{diff}}{D} = \frac{3}{d} \Leftrightarrow D = \frac{m_{diff} \cdot d}{3}$

$\underline{D_1} = \frac{1 \cdot 2{,}500 \cdot 10^{-6} m}{3} = \underline{\underline{0{,}833 \cdot 10^{-6} m}}$

$\underline{D_2} = \frac{2 \cdot 2{,}500 \cdot 10^{-6} m}{3} = \underline{\underline{1{,}667 \cdot 10^{-6} m}}$