Kont. eks. 29.8.89 Klassisk feltteori

1 a) $\mathcal{L} = \bar{\psi}\, c\left[\gamma^\mu\left(\frac{\hbar}{i}\partial_\mu - eA_\mu\right) - mc\right]\psi$

Varierer ψ og $\bar{\psi}$ hver for seg

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} - \frac{d}{dx^\mu}\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = \underline{\left[c\gamma^\mu\left(\frac{\hbar}{i}\partial_\mu - eA_\mu\right) - mc^2\right]\psi = 0} \qquad \text{da} \qquad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} = 0$$

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial \psi} &= \bar{\psi}\left[c\gamma^\mu(-eA_\mu) - mc^2\right] \\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} &= \bar{\psi}\, c\gamma^\mu \frac{\hbar}{i}\end{aligned}\right\} \begin{aligned}&-\bar{\psi}\, ec\gamma^\mu A_\mu - \bar{\psi}\, mc^2 - c\frac{\hbar}{i}\partial_\mu\bar{\psi}\gamma^\mu = 0 \\ &\underline{\left(c\left(-\frac{\hbar}{i}\partial_\mu - eA_\mu\right)\bar{\psi}\gamma^\mu - mc^2\bar{\psi}\right) = 0}\end{aligned}$$

b)

$$T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\partial_\nu\psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})}\partial_\nu\bar{\psi} - \delta^\mu{}_\nu \mathcal{L}$$

$$= \bar{\psi}\, c\gamma^\mu\frac{\hbar}{i}\partial_\nu\psi - \delta^\mu_\nu\left|\bar{\psi}\, c\gamma^\mu\left(\frac{\hbar}{i}\partial_\mu - eA_\mu\right)\psi\right. + \left.\delta^\mu_\nu\bar{\psi}\, mc^2\psi\right|$$

$$T^0{}_0 = \bar{\psi}\, c\gamma^0\frac{\hbar}{i}\partial_0\psi - \bar{\psi}\, c\gamma^0\left(\frac{\hbar}{i}\partial_0 - eA_0\right)\psi - \bar{\psi}\, c\gamma^k\left(\frac{\hbar}{i}\partial_k - eA_k\right)\psi + \bar{\psi}\, mc^2\psi$$

$$= \underline{\bar{\psi}\left[c\gamma^k\left(\frac{\hbar}{i}\partial_k - eA_k\right) + mc^2 + ec\gamma^0A_0\right]\psi} \qquad \begin{aligned}A_k &= -A^k = -\vec{A} \\ A_0 &= A^0 = \tfrac{1}{c}\phi \\ \gamma^k &= \beta\alpha^k \quad \gamma^0 = \beta\end{aligned}$$

$$= \underline{\psi^+\left[c\alpha^k\left(\frac{\hbar}{i}\partial_k - eA_k\right) + \beta mc^2 + e\phi\right]\psi}$$

Total energi når $\psi = 0$ utenfor volumet V

$$\underline{E = \int_V T^0{}_0\, d^3r}$$

Hamilton operatoren i Diraclikningen:

$$H = c\alpha^k\left(\frac{\hbar}{i}\partial_k - eA_k\right) + \beta mc^2 + e\phi$$

gir energien

$$\underline{E = \int \psi^+ H\psi\, d^3r} \qquad \text{som ovenfor}$$

Klassisk feltteori 29.8.89

2 a Lyset går korteste (ekstremal) vei i tidrommet

$$\delta\int ds = \delta\int\sqrt{\left(\frac{ds}{d\sigma}\right)^2}\,d\sigma = 0 \qquad \left(\text{eller også } \delta\int\left(\frac{ds}{d\sigma}\right)^2 d\sigma = 0\right)$$

med
$$\left(\frac{ds}{d\sigma}\right)^2 = \left(1-\frac{\varepsilon}{r}\right)c^2\left(\frac{dt}{d\sigma}\right)^2 - \frac{1}{1-\frac{\varepsilon}{r}}\left(\frac{dr}{d\sigma}\right)^2 - r^2\left[\left(\frac{d\vartheta}{d\sigma}\right)^2 + \sin^2\vartheta\left(\frac{d\varphi}{d\sigma}\right)^2\right]$$

Gir bevegelseslikningene

$$\frac{\partial\sqrt{\left(\frac{ds}{dt}\right)^2}}{\partial t} - \frac{d}{d\sigma}\frac{\partial\sqrt{}}{\partial\left(\frac{\partial t}{\partial\sigma}\right)} = 0 \quad\Rightarrow\quad -\frac{d}{d\sigma}\left(\frac{1}{\sqrt{}}\left(1-\frac{\varepsilon}{r}\right)c^2\frac{dt}{d\sigma}\right) = 0$$

$$\frac{\partial\sqrt{}}{\partial\varphi} - \frac{d}{d\sigma}\frac{\partial\sqrt{}}{\partial\left(\frac{\partial\varphi}{\partial\sigma}\right)} = 0 \quad\Rightarrow\quad -\frac{d}{d\sigma}\left(\frac{1}{\sqrt{}}r^2\sin^2\vartheta\frac{d\varphi}{d\sigma}\right) = 0$$

Gir
$$c\left(1-\frac{\varepsilon}{r}\right)\frac{dt}{d\sigma} = \text{konst} = A \quad\Rightarrow\quad \underline{\frac{dt}{d\sigma} = \frac{A/c}{1-\frac{\varepsilon}{r}}}$$

$$r^2\sin^2\vartheta\,\frac{d\varphi}{d\sigma} = \text{konst} = B \quad\Rightarrow\quad \underline{\underline{\frac{d\varphi}{d\sigma} = \frac{B}{r^2}}} \qquad \text{når } \vartheta = \frac{\pi}{2}$$

b) Fra intervallikningen som for en lysstråle $ds = 0$ er

$$\left(1-\frac{\varepsilon}{r}\right)c^2\left(\frac{dt}{d\sigma}\right)^2 - \frac{1}{1-\frac{\varepsilon}{r}}\left(\frac{dr}{d\sigma}\right)^2 - r^2\left(\frac{d\varphi}{d\sigma}\right)^2 = 0$$

får ved innsetting

$$\left(1-\frac{\varepsilon}{r}\right)c^2\frac{\frac{1}{c^2}A^2}{\left(1-\frac{\varepsilon}{r}\right)^2} - \frac{1}{1-\frac{\varepsilon}{r}}\left(\frac{dr}{d\sigma}\right)^2 - r^2\frac{B^2}{r^4} = 0$$

$$\left(\frac{dr}{d\sigma}\right)^2 = A^2 - \frac{B^2}{r^2}\left(1-\frac{\varepsilon}{r}\right) = \left(\frac{dr}{d\varphi}\frac{d\varphi}{d\sigma}\right)^2 = \left(\frac{dr}{d\varphi}\right)^2\frac{B^2}{r^4} =$$

$$= B^2\left(\frac{du}{d\varphi}\right)^2 \qquad \text{med } u = \frac{1}{r} \qquad \frac{du}{d\varphi} = -\frac{1}{r^2}\frac{dr}{d\varphi}$$

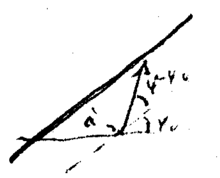
eller
$$\underline{\underline{\left(\frac{du}{d\varphi}\right)^2 = \left(\frac{A}{B}\right)^2 - u^2(1-\varepsilon u)}}$$

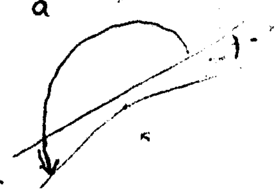
c) Derivasjon av banelikningen $\frac{d}{d\varphi}$ og divisjon med $\frac{du}{d\varphi}$ (når $\frac{du}{d\varphi} \neq 0$)

$$\underline{\frac{d^2u}{d\varphi^2} = -u + \frac{3}{2}\varepsilon u^2}$$

Løsning: 1. approks. $\varepsilon = 0$. $u = \frac{1}{r} = \frac{1}{a}\sin(\varphi-\varphi_0) \Rightarrow r\sin(\varphi-\varphi_0) = a$

Rett linje med retning φ_0 og avstand a fra origo.





2. approks.: $\frac{d^2u}{d\varphi^2} + u = \frac{3}{2}\varepsilon\frac{1}{a^2}\sin^2(\varphi-\varphi_0)$

$\pi + \frac{\varepsilon}{a}$

Løsning: $u = \frac{1}{a}\sin(\varphi-\varphi_0) + \frac{3}{4}\frac{\varepsilon}{a^2}\left(1+\frac{1}{3}\cos 2(\varphi-\varphi_0)\right)$

Velger $\varphi_0 = 0$. Banen ligger under den rette linjen: $a - r\sin\varphi = \frac{3}{4}\frac{\varepsilon}{a}r\left(1-\frac{1}{3}\cos 2\varphi\right) > 0$

Langt borte $r \to \infty$ $u \to 0$ (φ_∞ liten $\cos 2\varphi_\infty \approx 1$)

$$0 \approx \frac{\sin\varphi_\infty}{a} + \frac{3\varepsilon}{4a^2}\left(1+\frac{1}{3}\right) \qquad \sin\varphi_\infty \approx -\frac{\varepsilon}{a} \qquad \varphi_\infty \approx -\frac{\varepsilon}{a} \quad\text{eller}\quad \varphi_\infty \approx \pi + \frac{\varepsilon}{a}$$

Avbøyning: $\Delta\varphi = 2|\varphi_\infty| = \underline{\underline{\frac{2\varepsilon}{a}}}$