EXAM IN FY 2290 Energy resources **SUGGESTED SOLUTUION**
June 5th 2010

PROBLEM 1: LOGISTIC CONSUMPTION MODEL (15%)

a) (5%) *In a mathematical model for the consumption of a limited resource (such as fossil fuels), the variation of the consumption P as a function of time can be expressed as*

$$P(t) = \frac{1}{\beta}\left(1 - \frac{Q(t)}{Q_\infty}\right)Q(t).$$

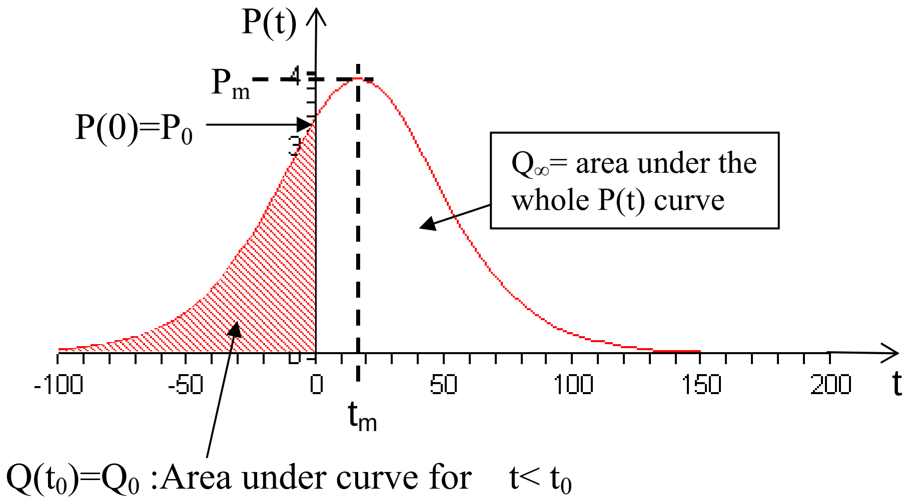
What does β, Q(t) and Q_∞ represent in this expression?

Q(t) is the accumulated consumption up to a certain time t,
Q_∞ is the total amount of the resource, and
β is a constant

What assumptions have been made about the relationship between P(t) and Q(t) and Q_∞? Are these realistic assumptions? Why/why not?

One assumes that P(t) is proportional to how much has been consumed of the resource up till time t ; Q(t), and to how much is left of the resource at that time; $\frac{Q_\infty - Q(t)}{Q_\infty}$. These are realistic assumptions for the consumption of limited resources: In the beginning, when a lot of the resource is remaining, you will consume a lot, but as the remaining amount of the resource reduces, the consumption will also be reduced.

b) *(5%) Make a sketch of P(t) as a function of time and indicate $Q(t_0)$ for a given time t_0. Also indicate Q_∞ in the figure.*
Indicate at what time the consumption is largest. How much remains of the resource at that time?



The consumption is largest at $t=t_m$. At this time half of the resource remains.

c) *(5%) In BP's energy statistics the ratio R/P is used to say something about how long a resource will last. Explain what R/P is.*
Explain why it may change form year to year.
Approximately how large is R/P for oil, natural gas and coal at present?

R is the reserves of a resource at the end of a specific year, and P is the production of that resource that year. If we assume that the consumption equals the production, then R/P equals the number of years that a resource will last, if R and P are constant.

R/P may change from year to year since R may change, i.e. how much of the resource is known and economical to extract. Also P will change.

R/P is at present ca 41 years for oil, 67 years for gas and 164 years for coal.

PROBLEM 2: THE GREENHOUSE EFFECT (20%)

a) *(10%) What is the natural greenhouse effect? (The human made greenhouse effect comes in problem b)*
Which gases contribute the most to the natural green house effect?
What would the temperature on Earth have been if there was no atmosphere on Earth?

The atmosphere is (quite) transparent for the incoming (short wavelength) solar radiation, and this radiation ends up at the surface of the earth where it is partially absorbed and reflected. The surface heats up due to the absorption. The surface will emit long wavelength thermal radiation due to its temperature, into the atmosphere. In the atmosphere of the earth there are gasses present (water vapour, CO_2 and others) that will absorb parts of the long wavelength (thermal) radiation emitted from the surface of the earth. In this way the temperature of the atmosphere increases and the amount of black body radiation emitted back to the surface of the earth increases. A fraction of the long wavelength radiation is thus re-emitted back to the surface and causes a temperature increase here.

The atmosphere thus acts similar to the glass in ordinary greenhouses: letting short wavelength radiation in to heat up the interior, while keeping the long wavelength radiation on the inside.

For the natural greenhouse effect, water vapour and CO2 has the largest contribution.

Without the atmosphere there would be no greenhouse effect and the temperature would be ca 30K lower than it is.

b) *(5%) How does human activities contribute to the greenhouse effect?*
List the most important greenhouse gasses related to this, and describe shortly what one can do to reduce the emissions.

Human activities enhance the greenhouse effect, and the most important human induced greenhouse gases are CO_2, N_2O, CH_4 and halocarbons. The emissions are strongly related to consumption of fossil fuels (especially CO2), and the solution to the reduction of greenhouse gas emissions is to reduce the use of these fuels. This can be done by replacing them with renewable energy sources or nuclear energy, energy conservation and possibly by capture and storage of CO2 (CCS).

c) *(5%) IPCC has estimated how much warmer it will get in Earth for various scenarios and models. How warm does it get in worst case, according to these calculations? List three examples of the effect of global warming.*

It is estimated that the temperature increase will be in the range 2.1 and 4.6K in year 2100. In the worst case the temperature increase will be ca 6K.

Examples of the effect of global warming are

- more extreme weather
- melting of ice
- changes in ecosystems due to the temperature increase
- spreading of deceases
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PROBLEM 3: HEAT ENGINES (20%)

a) *(5%) What is a heat engine?*
What can it be used for?
Where can the heat be taken from?

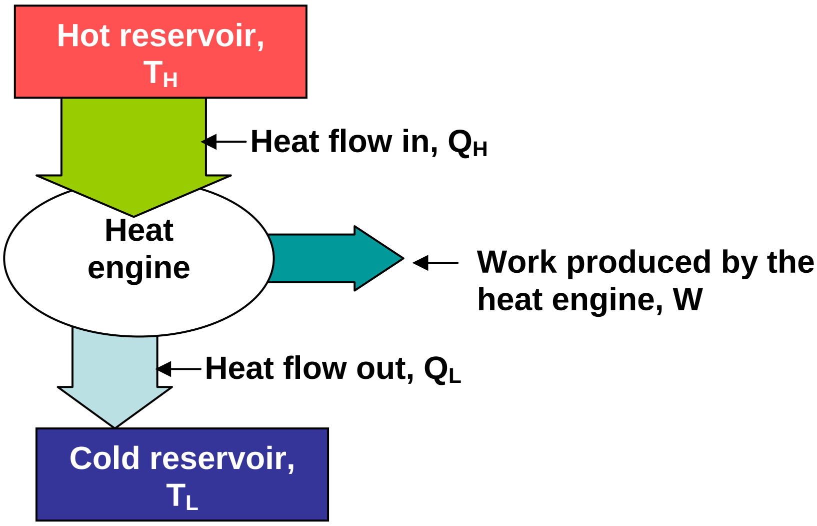
A heat engine is a device that converts heat to work. Heat engines are generally used in transportation (gasoline engines, diesel engines, jet engines) and to produce electricity (steam turbines, gas turbines), but can also be used to produce mechanical work for other purposes than transportation.
Examples of heat sources are

- Fossil fuels (coal, oil, natural gas)
- Biofuels (wood, ethanol, methane)
- Solar energy
- Geothermal energy (from geological "hot spots")
- Ocean thermal energy

b) *(10%) How can one express the efficiency of a heat engine used for electricity production?*
Draw a thermodynamic diagram for the heat engine.
What limits the efficiency of such heat engines?

One can express the efficiency as $efficiency = \frac{electricity\_out}{energy\_input}$, or $\eta = \frac{W_{el}}{Q}$. Where W_{el} is the amount of electricity generated, and can be expressed as the work generated by the heat engine W multiplied by the efficiency of the generator converting the work into electricity η_{gen}.

Thermodynamic diagram of the heat engine:



The efficiency of the heat engines is limited by the fact that not all the heat supplied (Q_H) can be converted to work (2^{nd} law of thermodynamics). Some of the heat (Q_L) has to be given off to a cold reservoir and this limits the amount of work and thus the efficiency.

c) *(5%) What is the upper limit for the efficiency of a gas fired power plant, if we assume that the gas burns at 1500 °C and that the exhaust from the turbine has a temperature of 1000 °C?*
How can the energy in the gas be better utilised?

The limiting efficiency (the Carnot efficiency) can be expressed as

$$\eta = 1 - \frac{T_L}{T_H}$$

where T_L = 1000C= 1273K and T_H =1773K in this case:

$$\eta = 1 - \frac{1273}{1773} = 0.28 = 28\% .$$

The energy in the gas can be better utilised if the hot exhaust is used to run a steam turbine, and if also the waste heat from the steam turbine is used as a heat source, e.g. for process heat or heating of building, in a combined heat and power plant (CHP) with two heat engines (both a steam and a gas turbine).

PROBLEM 4: ELECTRICITY SUPPLY (45%)

a) *Hydro power (10%)*
The annual electricity consumption in Norway is ca 110TWh per year. (1 TWh = 10^{12}Wh). If the consumption increases by 10%, how large area of a 50m deep water reservoir is needed to supply this additional amount of electricity from a hydro power plant with generator efficiency of 95%?

Assume 100m height difference from reservoir to the turbine, that the reservoir is emptied once in a year and that the density of water is ρ=1000kg/m^3.

The volume of water in the reservoir equals $V = A \times d$, where A is the area we want to calculate and *d* the depth. We assume that all the potential energy of the water is converted to kinetic energy in the turbine:

$$E_{kin} = E_{pot} = mgh = \rho Vgh = \rho(A \cdot d)gh$$

This kinetic energy is converted to electric energy with an efficiency η = 95%:

$$E_{el} = E_{kin} \cdot \eta$$

We insert for E_{kin} and solve for A:

$$E_{el} = E_{kin} \cdot \eta = \rho(A \cdot d)gh \cdot \eta$$

$$\Updownarrow$$

$$A = \frac{E_{el}}{\rho \cdot d \cdot g \cdot h \cdot \eta}$$

where E_{el} is the increase in electricity consumption (10% of 110TWh):

$$E_{el} = 10\% \cdot 110TWh = 11TWh = 11 \cdot 10^9 kWh = 11 \cdot 10^9 \cdot 3.6 \cdot 10^6 J = 3.96 \cdot 10^{16} J$$

We insert the numerical values in the expression for A and get:

$$A = \frac{E_{el}}{\rho \cdot d \cdot g \cdot h \cdot \eta} = \frac{3.96 \cdot 10^{16} J}{1000kg/m^3 \cdot 50m \cdot 9.8m/s^2 \cdot 100m \cdot 0.95} = 8.5 \cdot 10^8 \frac{J}{kg/s^2}$$

$$= 8.5 \cdot 10^8 m^2 = 850km^2$$

since 1 $J = 1\ kgm^2/s^2$ and $10^6\ m^2$ = 1 km^2.

b) <u>*Electricity from solar cells (15%)*</u>
How does a solar cell work?

In a solar cell solar radiation is converted to electric energy by absorption of photons in a semiconductor. Only photons with energy larger than the bandgap can be absorbed; the rest are transmitted. In the absorption process an electron is excited from a state in the valence band (where the electron takes part in the bonding of the semiconductor) to a state in the conduction band (where the electron is able to conduct a current through the semiconductor). The empty state left behind in the valence band is called a hole. To reduce the recombination of the electrons and holes, a built-in electric field is formed by making a pn-junction in the solar cell. The electrons and holes will move in opposite directions in this built-in electric field since they have opposite charge. (The electrons move from the p-side to the n-side.) Outside the pn-junction, the electrons and holes will move to the external contacts on the solar cell. At the contact on the n-side of the solar cell pn-junction, the electrons will have a higher (electrostatic) potential energy than (the holes) on the p-side, and this potential difference gives rise to the voltage over the solar cell (called the photo-voltage). This voltage forces the photogenerated electrons (on the n-side) out of the cell and through the load where they give up their energy, before they return into the solar cell at the p-side, at the lower potential energy. In summary: electrons inside the solar cell get an increased potential energy by the absorption of photons, and this increased potential energy (which is partially lost inside the solar cell) gives rise to a voltage over the cell and a current (made up by the photogenerated electrons) will flow out of the cell and through the external circuit.

How large area of Norway need to be covered with solar cells to generate 11TWh electric energy in one year, if the annual solar irradiation is 900kWh/m² and the solar cell has a typical efficiency. (If you don't remember the typical efficiency, assume that it is 20%).

The amount of solar electricity generated by the solar cells can be expressed as

$E_{PV} = E_{in} \cdot \eta_{PV} \cdot A_{PV}$

where E_{in} is the incoming solar energy, η_{PV} the solar cell efficiency, typically 15% and A_{PV} is the area of the solar cells, that we want to calculate. To generate 11TWh the area needed is found by setting $E_{PV} = 11TWh$ and solving for A_{PV}:

$$A_{PV} = \frac{E_{PV}}{E_{in} \cdot \eta_{PV}} = \frac{11TWh}{900kWh/m^2 \cdot 0.15} = 8.15 \cdot 10^7 m^2 = 81km^2$$

For a solar cell with an efficiency of 20%:

$$A_{PV} = \frac{E_{PV}}{E_{in} \cdot \eta_{PV}} = \frac{11TWh}{900kWh/m^2 \cdot 0.20} = 6.11 \cdot 10^8 m^2 = 61km^2$$

c) <u>*Electricity from bioenergy (15%)*</u>
What is bioenergy? What are the advantages and problems related to using bioenergy?

Bioenergy is energy extracted from organic materials, such a plants, plant oils, waste and residues from food industry. The main advantage is that bioenergy potentially is CO2 neutral in that the plants capture the same amount of CO2 as they will release when the biomaterial is burnt. The production of bioenergy can have many advantages if conducted in the correct way, with increased usage of waste, increased number of jobs etc, but may also lead to competition with food production, ersoision etc.

Assume that the forests in Norway can supply 100 $GJha^{-1}$ per year, how many km^2 is then needed to generate 11TWh electric energy per year in a steam turbine power plant fuelled with timber from the forests? Assume a steam turbine power plant efficiency of 36% (including the generator efficiency). 1 ha = 10^4 m^2.

The amount of bioenergy produced per forest area is given in GJ/ha and we need to convert this to TWh/km^2. 1J equals 1/3600Wh, 1TWh= 10^{12}Wh, and 1ha=$10^4$$m^2$=$10^{10}$$km^2$, so that

$$E/A = 100GJ/ha = 100 \cdot 10^9 \cdot \frac{1}{3600} \cdot Wh/ha = 2.8 \cdot 10^7 Wh/ha = 2.8 \cdot 10^7 Wh/10^4 m^2$$
$$= 2.8 \cdot 10^3 Wh/m^2 = 2.8 \cdot 10^9 Wh/km^2 = 2.8 \cdot 10^{-3} TWh/km^2$$

For a steam turbine power plant with efficiency η_{steam}, the electric energy E_{el} generated from an incoming amount of bioenergy of E_{in} (assuming that all the energy in the bio mass is transferred to the steam boiler) can be expressed as

$E_{el} = E_{in} \cdot \eta_{steam}$

For a forest of area A_{forest} the total energy produced is $E_{in} = E/A \cdot A_{forest}$, we insert this in the expression for E_{el}, solve for A_{forest} and find:

$$E_{el} = E_{in} \cdot \eta_{steam} = E/A \cdot A_{forest} \cdot \eta_{steam} \Leftrightarrow A_{forest} = \frac{E_{el}}{E/A \cdot \eta_{steam}}$$

The amount of electric energy we want to generate equals 11TWh, so that

$$A_{forest} = \frac{E_{el}}{E / A \cdot \eta_{steam}} = \frac{11TWh}{2.8 \cdot 10^{-3} TWh / km^2 \cdot 0.36} = 1.1 \cdot 10^4 km^2$$

d) <u>Coal fired power plants (5%)</u>
What is the typical efficiency of a coal fired power plant? If the same amount of electricity (11TWh/year) should be provided by a typical coal fuelled power plant, approximately how much thermal energy should the coal provide (in TWh per year)? (Use 40% for the efficiency if you don't know what it typically is.) How much coal would this correspond to if all the chemical energy stored in the coal is converted to heat in the power plant?

What are the advantages for coal fired power plants compared to hydro power, solar cells and bioenergy?

The typical efficiency is 30-40%. (World average is 31%, and the world record is approaching 50%. BP uses 38% in their report.)

Using an efficiency of $\eta_{coal} = 38\%$, the amount of thermal power E_{th} from the burning of the coal should equal $E_{th} = \frac{E_{el}}{\eta_{coal}}$, where E_{el} is the amount of electric energy to be generated, i.e. $E_{th} = \frac{E_{el}}{\eta_{coal}} = \frac{11TWh}{0.38} = 28.9TWh$.

For en energy density of 7800 kWh/907 kg = 8.6kWh/kg, the amount of coal needed is

$$m_{coal} = \frac{28.9TWh}{8.6kWh / kg} = \frac{28.9 \cdot 10^{12} Wh}{8.6 \cdot 10^3 Wh} kg = 3.3 \cdot 10^9 kg = 3.3 \cdot 10^6 tonne$$

The advantages for coal fired power plants compared to

- hydro power is that electricity can be generated continuously (hydro power depends on the water supply)
- solar cells is that electricity can be generated continuously, that the cost of the electricity is low and that the technology is established
- bioenergy is that the energy density is higher, so that less amount of fuel is needed to generate the same amount of electricity.