EKSAMEN I EMNE FY3006 Målesensorer/transdusere

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Model Answers

Problem 1

Part a:

1. The kinetic energy of an electron which leaves the photocathode with zero speed and arrives at the first dynode is evidently eV_{K1}. If N_t secondary electrons are emitted, an amount of energy equalling $w_0 N_t$ will be spent in overcoming the work function, and the rest will appear as the kinetic energy of the electrons ejected from the first dynode. Thus energy conservation gives

$$eV_{K1} = w_0 N_t + \int_0^\infty wN(w)\,\mathrm{d}w = 2w_0 N_t, \tag{101}$$

which can be rearranged as Eq. (3) of the examination paper.

2. The total kinetic energy is given by the integral on the right-hand side; thus $\overline{E} = w_0$

3. The time (Δ) taken by a photoelectron to travel from its point of ejection to the collecting anode is called *electron transit time*. The four assumptions imply that $\Delta = N\delta$, where δ is the time taken to travel the distance (s) from one stage to the next. We are dealing with a case of uniformly accelerated motion under the action of a force $F = eE$, where $E = U/s$ is the electric field intensity. The acceleration is $a = F/m$ and $s = a\delta^2/2$, so that

$$\delta = \sqrt{\frac{2s}{\rho E}} = s\sqrt{\frac{2}{\rho U}} = sf(\rho U). \tag{102}$$

4. A look at Fig. 1 shows that the total distance between K and A cannot be very much larger than 50 mm. Accordingly we may take $s = 5\times10^{-3}$. We also have $\sqrt{10^{-11}\times10^{-2}} \approx 3\times10^{-7}$. This gives $\delta \approx 1.5$ ns, and $\Delta \approx 15$ ns.

5. The value of w_0 is close to 1 eV. Thus the average kinetic energy of an ejected electron will also be close to 1 eV; on the other hand, the energy of the electron at the end of its trajectory is 100 eV. This means that assumptions a and b do not introduce large errors in our estimate of Δ.

Part a:

1. The drift velocities of the electrons and positive ions in an ionization chamber are vastly different. The fast electrons rapidly approach the anode while the positive ions drift slowly towards the cathode, and set up a space charge which in turn modifies the collection field of the electrons and thus the electron collection time. If the electric field is uniform, the performance of the detector is severely degraded. A simple way of overcoming the positive ion space charge problem is to use the cylindrical geometry, which provides a highly non-uniform field. The electrons approaching the wire anode are now in a rapidly increasing electric field and the positive ion space charge exerts little effect.

2. The Lyman-α wavelength is 121.6 nm. A convenient choice would be an NO-filled ionization chamber with a MgF_2 window. Detectors of this type are sensitive only to wavelengths between 115 nm, where the window begins to transmit and 135 nm, corresponding to the lowest photon energy capable of ionizing the filling gas.

3. Owing to their large mass (compared with that of an electron), the positive ions of Ar (the major component of the filling vapour) move very little while the electron avalanche is being collected by the anode. These cations will eventually approach the cathode, and become neutralized by capturing an electron. During the long travel time of the cations, the counter will not be able to respond to the arrival of a new particle. But there is an even more important point to consider: When an Ar^+ cation pulls an electron from the cathode, the end result is an Ar atom that is neutral but in a highly excited electronic state. This excited atom will subsequently decay by emitting an ultraviolet photon, which can itself give rise to the ejection of a photoelectron from the surface of the cathode. Now, if photoelectrons are liberated following the arrival of Ar^+ cations near the cathode, the whole process will start all over again, producing an endless continuous discharge that would render the detector useless. An early method for preventing this endless cycle used external circuitry to "quench" the tube; instead of this active quenching method, a passive method, involving the introduction of organic or halogen vapours, is now preferred. The complex molecule of the quenching vapour is selected to have a lower ionization potential (< 10 eV) than that of the fill gas (26.4 eV). Upon collision with a vapour molecule the fill gas ion gives up 10 eV to the quench vapour molecule which then quickly dissociates instead of losing its energy by radiative emission. The remainder of the partially neutralized vapour-atom energy (4 eV) produces an ultraviolet photon that is strongly absorbed by the molecules and prevented from striking the cathode. Any quenching gas that might be accelerated and hit the cathode dissociates upon contact.

Problem 2

1.

$$S = k_{\mathrm{ET}}[X^*][Y] \tag{6}$$

$$\alpha = k_{\mathrm{RX}} + k_{\mathrm{NX}} + k_{\mathrm{ET}}[Y] \tag{7}$$

$$\beta = k_{\mathrm{RY}} + k_{\mathrm{NY}} \tag{8}$$

2. Let us first introduce some symbols as labour-saving devices:

$$x = [X^*], \quad y = [Y^*], \quad k = k_{\mathrm{ET}}[Y], \tag{201}$$

$$B_0 = AI_0, \tag{202}$$

$$D = \mathrm{d}/\,\mathrm{d}t. \tag{203}$$

We substitute these expression into the rate equation, and rearrange them as follows :

$$(D + \alpha)x = B_0 + B_0 m_0 \cos\omega t \tag{204}$$

$$(D + \beta)y = kx \tag{205}$$

We are interested in the particular integrals only, which will be denoted by $\tilde{x}$ and $\tilde{y}$.

$$\tilde{x} = \frac{B_0 + B_0 m_0 \cos\omega t}{(D + \alpha)} \tag{206}$$

$$= \frac{B_0}{(D + \alpha)} + \frac{B_0 m_0 \cos\omega t}{(D + \alpha)} \tag{207}$$

$$= \frac{B_0}{\alpha} + \frac{B_0 m_0}{\sqrt{\omega^2 + \alpha^2}} \cos(\omega t - \phi_X) \tag{208}$$

$$= \frac{B_0}{\alpha} + \frac{B_0 m_0}{\alpha} \lambda_X \cos(\omega t - \phi_X) \tag{209}$$

$$= \frac{B_0}{\alpha}\left[1 + m_X \cos(\omega t - \phi_X)\right], \qquad m_X \equiv m_0 \lambda_X. \tag{210}$$

$$(D + \beta)\tilde{y} = k\tilde{x} \quad \Longrightarrow \quad \tilde{y} = \frac{k\tilde{x}}{D + \beta} \tag{211}$$

$$\tilde{y} = \frac{kB_0}{\alpha}\left[\frac{1 + m_0 \lambda_X \cos(\omega t - \phi_X)}{D + \beta}\right] \tag{212}$$

$$= \frac{kB_0}{\alpha}\left[\frac{1}{\beta} + \frac{m_0 \lambda_X \lambda_Y \cos(\omega t - \phi_X - \phi_Y)}{\beta}\right] \tag{213}$$

$$= \frac{kB_0}{\alpha\beta}\left[1 + m_0 \lambda_X \lambda_Y \cos(\omega t - \phi_X - \phi_Y)\right] \tag{214}$$

$$= B_1[1 + m\cos(\omega t - \phi)]. \tag{215}$$

$$\text{where} \quad B_1 \equiv kB_0/(\alpha\beta), \quad m \equiv m_0 \lambda_X \lambda_Y, \quad \phi \equiv \phi_X + \phi_Y. \tag{216}$$

Eq. (12) of the examination paper now follows, since $F_Y = k_{\mathrm{RY}}\tilde{y}$.

3. We will need an apparatus for analyzing the fluorescence emission spectra, whose major components are a monochromator and a fast-response photomultiplier tube (PMT). The output of the PMT will be fed to the signal channel of the lock-in amplifier. Even if we set the monochromator to pass a wavelength where only the acceptor Y^* emits, we can only measure the sum $\phi \equiv \phi_X + \phi_Y$. In order to measure ϕ_X, we must set the monochromator to a wavelength where only the donor X^* emits. Our phase measurements will be reliable only if the phase angle is close to 45 degrees, because this is where the change in the tangent is most sensitive to the phase angle. The modulation frequency must be chosen so as to satisfy this condition.

4. Since ϕ_Y can only be found by subtracting ϕ_X from ϕ; this means that reliable measurements of ϕ_Y can only be made when it is close to ϕ_X.

5. Triplet state lifetimes are much longer than the lifetimes of singlet excited states. Since the modulation frequency is high, triplet concentrations would attain time-independent values, and will not be detected by the lock-in amplifier.

Problem 3 *Part a:*

1. The bridge is balanced initially when $M_0 = N_0$, $P = Q$, and $R = S$, and

$$(M_0 + P)S = (N_0 + Q)R \tag{13}$$

If the beam is now bent, so that P and R undergo strain ϵ and Q suffers a strain $-\epsilon$, and the bridge is balanced again, keeping $M = N$, then

$$[M + P(1 + \epsilon g)]S = [N + Q(1 - \epsilon g)]R(1 + \epsilon f). \tag{14}$$

2. Dividing Eq. (14) by Eq. (13), putting $M = N$, $P = Q$, and solving for f, one easily arrives at the result

$$f = \frac{2Qg}{N + Q(1 - \epsilon g)}, \tag{15}$$

which reduces, when one ignores the product ϵg to the result shown below:

$$f = \frac{2Qg}{N + Q}, \tag{16}$$

3. Eq. (16) can be justified by recalling that g is usually close to 2 for metal wire strain gauges and $\epsilon \approx 10^{-3}$. This means that $\epsilon g \ll 1$, and we can write

$$f = \frac{2Qg}{N + Q}\left(1 + \frac{\epsilon}{2}\frac{2Qg}{N + Q},\right),$$

and by replacing $2Qg/(N + Q)$ by f, we obtain

$$f = \frac{2Qg}{N + Q} + \frac{\epsilon f^2}{2}, \tag{17}$$

which shows that, in using Eq. (16), we make an error of $\epsilon f^2/2$. Eq. (16) is very useful because its right hand side doe not depend on ϵ, and is in fact a function of N alone, Q and g being constant for a particular arrangement.

Part b:

1. It is shown on the next page that the combination of resistors can be viewed as a potential divider, and one finds

$$M = \frac{UW}{U + V + W}, \tag{18}$$

$$N = \frac{VW}{U + V + W}. \tag{19}$$

2. If one substitutes for N in Eq. (16) from Eq. (19), and puts $U = V$, the result shown in Eq. (20) of the examination paper immediately follows.

We will now examine the two versions of the bridge shown below.

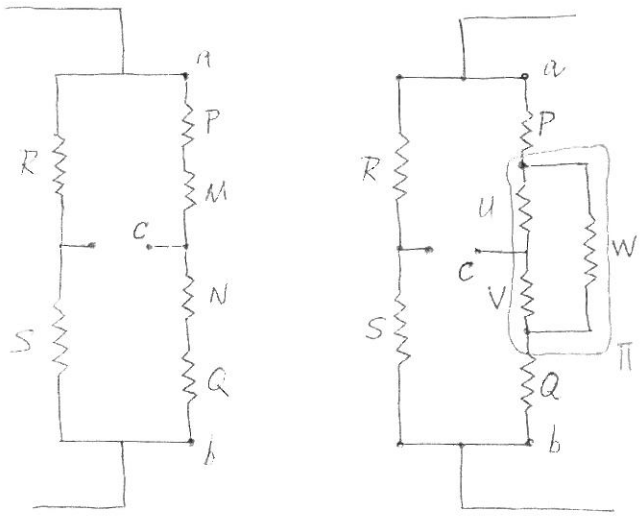

Figure 1: The original (left) and the modified (right) Wheatstone bridge

We will denote, in each bridge, the resistance between points x and y by R_{xy}, where $(x, y) = (a, b), (b, c)$ or (a, c). Let us consider the original bridge first, and note that

$$R_{ac} = P + M, \qquad R_{bc} = N + Q, \tag{301}$$

$$R_{ab} = P + Q + (M + N). \tag{302}$$

We now consider the modified bridge: let Π denote the resistance of the combination enclosed within the loop, and put $\Sigma = U + V + W$ for convenience. Obviously,

$$\Pi = W||(U + V) = \frac{W(U + V)}{U + V + W} = \frac{UW}{\Sigma} + \frac{VW}{\Sigma} \tag{303}$$

In the modified bridge

$$R_{ac} = P + \frac{UW}{\Sigma} + \frac{UV}{\Sigma}, \qquad R_{bc} = Q + \frac{VW}{\Sigma} + \frac{UV}{\Sigma}. \tag{304}$$

$$R_{ab} = P + Q + \underbrace{\overbrace{\frac{UW}{\Sigma}}^{M} + \overbrace{\frac{VW}{\Sigma}}^{N}}_{\Pi}. \tag{305}$$

If we compare Eq. 305 with Eq. 302 that the two circuits may be regarded as equivalent if we set

$$M = \frac{UW}{\Sigma}, \qquad M = \frac{VW}{\Sigma}. \tag{306}$$

If we compare Eq. 304 with Eq. 301 , we will see that we no longer have $R_{ab} = R_{ac} + R_{bc}$: an extra resistance gets added to the balancing arm, but finding its value was not a part of the exercise.