FY3464 KVANTEFELTTEORI I

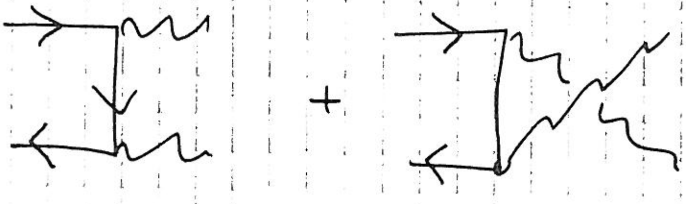
Løsning eksamen 5.12.2008

Oppgave 1.

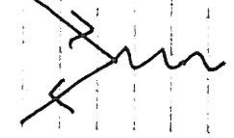
a) $\mu^- \to e^- \gamma$

Umulig i QED. Bryter μ- og e-tall. (Forventes å skje i naturen, men aldri observert)

b) $e^- e^+ \to \gamma\gamma$

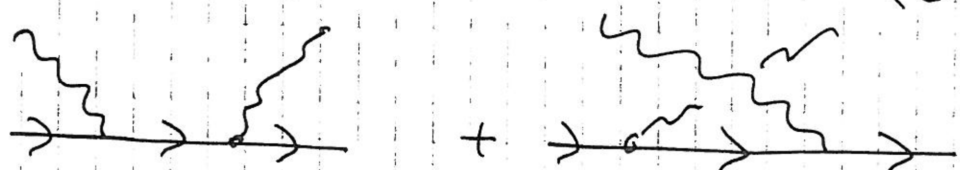


c) $e^- e^+ \to \gamma$



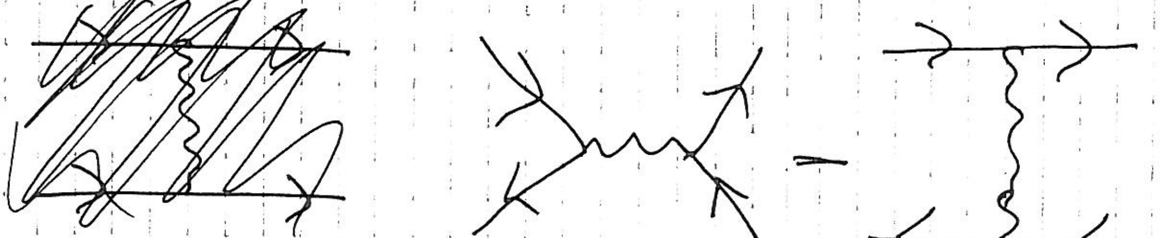
Men kinematisk umulig pga bevaring av 4-impuls.

d) $e^- \gamma \to e^- \gamma$ (Compton)

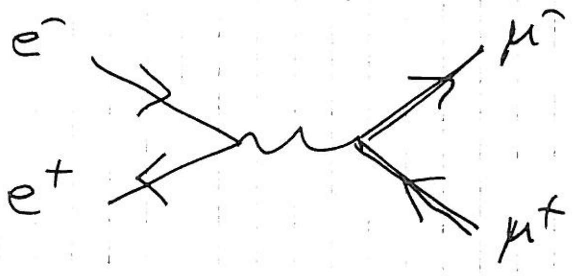


e) $e^+ \gamma \to e^+ \gamma$ (Compton)

f) $e^- e^+ \to e^- e^+$ (Babha)



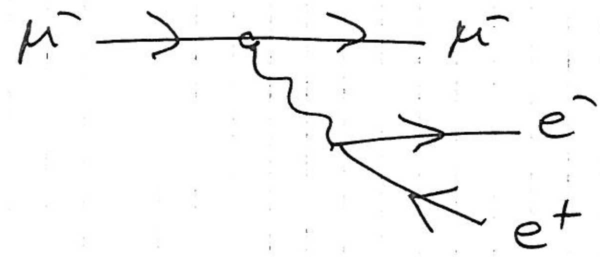
g) $e^-e^+ \to \mu^-\mu^+$



h) $e^-\mu^+ \to e^+\mu^-$

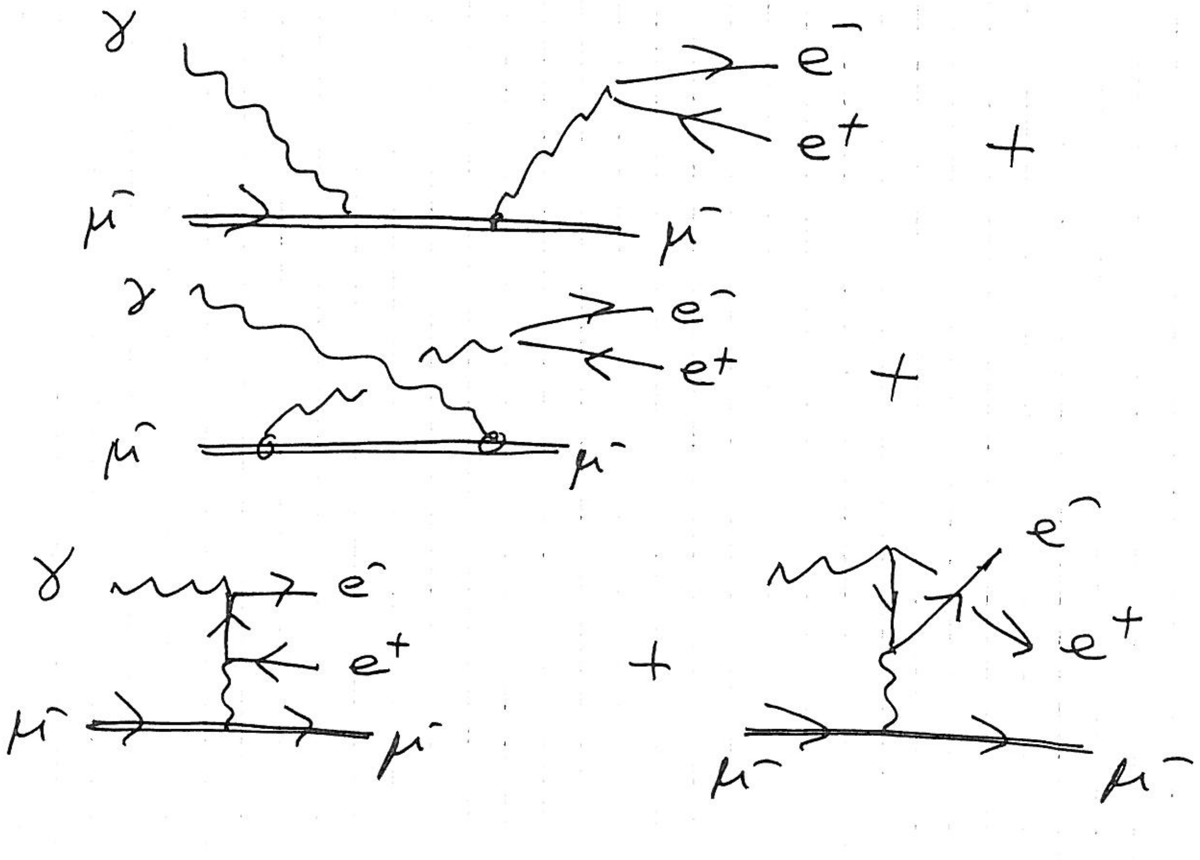
Umulig i QED.
Bryter e- og μ-tall.

i) $\mu^- \to \mu^- e^- e^+$



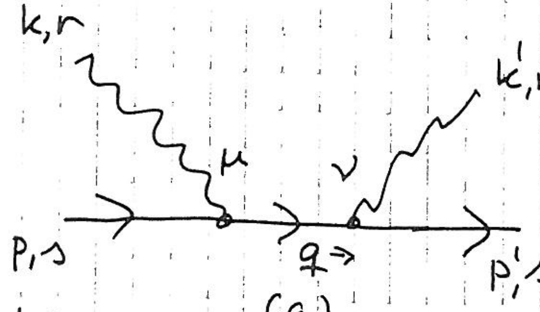
Men kinematisk umulig pga bevaring av 4-impuls.

j) $\mu^-\gamma \to \mu^- e^- e^+$



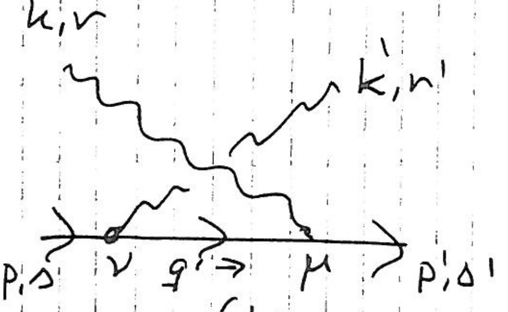
Oppgave 2

a) $e^- \gamma \to e^- \gamma$



(a)

+



(b)

b)

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)}$$

$$-i\mathcal{M}_{fi}^{(a)} = (ie)^2 \frac{i}{q^2-m^2} \; \bar{u}(p',s')\, \not{e}' (\not{q}+m) \not{e}\, u(p,s)$$

$$-i\mathcal{M}_{fi}^{(b)} = (ie)^2 \frac{i}{q'^2-m^2} \; \bar{u}(p',s')\, \not{e} (\not{q}'+m) \not{e}'\, u(p,s).$$

Her ~~er~~ $\not{e} \equiv \gamma_\mu e_r^\mu(k)$, $\not{e}' \equiv \gamma_\nu e_{r'}^\nu(k')$,

$\not{q} = \not{p} + \not{k}$, $\not{q}' = \not{p} - \not{k}'$.

c) $p = (E, 0, 0, -\omega)$, $\quad E = \sqrt{m^2+\omega^2}$

$p' = (E, -\omega \sin\vartheta, 0, -\omega\cos\vartheta)$.

I massesentersystemet er den totale 3-impulsen lik null. Dette bestemmer $\vec{p}$ og $\vec{p}'$; p^0 og p'^0 finnes så av energi-impuls relasjonen.

d) $e_\perp = (0,0,1,0)$ $\qquad e_\parallel = (0,1,0,0)$

$e'_\perp = (0,0,1,0)$ finnes ved triviell inspeksjon.

Som første forsøk setter vi

$\tilde{e}'_\parallel = (0, \cos\vartheta, 0, -\sin\vartheta)$ (som oppfyller $k' \tilde{e}'_\parallel = 0$)

Oppgave 2 forts.

④

d forts)

Men $(\tilde{e}'_{\parallel} p) = \tilde{e}'^{\mu}_{\parallel} p_{\mu} = -\omega \sin\vartheta$, så dette er ikke helt rett.

Velger derfor

$$e'^{\mu}_{\parallel} = \tilde{e}'^{\mu}_{\parallel} - c\, k'^{\mu}$$

med

$$c = \frac{(\tilde{e}' p)}{(k' p)} = \frac{-\omega \sin\vartheta}{\omega E + \omega^2 \cos\vartheta}$$

$$= -\frac{\sin\vartheta}{E + \omega\cos\vartheta}$$

Fordi $(k' e'_{\parallel}) = 0$ og $k'^2 = 0$ har vi at

$$e'_{\parallel}\, e'_{\parallel} = \tilde{e}'_{\parallel}\, \tilde{e}'_{\parallel} = -1.$$

Videre er

$e'_{\parallel}\, e'_{\perp} = 0$ fordi verken $\tilde{e}'_{\parallel}$ eller k' har noen komponent i y-retningen.

e) I $\mathcal{M}^{(a)}_{fi}$ kan vi skrive

$$T^{(a)} \equiv \bar{u}(p',s')\, \not{e}' (\not{q} + m)\, \not{e}\, u(p,s)$$

$$= \bar{u}(p',s')\, \not{e}' (\not{p} + \not{k} + m)\, \not{e}\, u(p,s)$$

$$= \bar{u}(p',s')\, \not{e}' \not{k} \not{e}\, u(p,s) - \bar{u}(p',s')\, \not{e}' \not{e} (\not{p} - m) u(p,s)$$

fordi $\not{p}\not{e} = -\not{e}\not{p}$ når $(ep) = 0$.

Videre oppfyller $u(p,s)$ Dirac-ligningen, $(\not{p} - m)u(p,s) = 0$, så vi får

$$\underline{T^{(a)} = \bar{u}(p',s')\, \not{e}' \not{k} \not{e}\, u(p,s)}$$

Tilsvarende fås $T^{(b)} \equiv \bar{u}(p',s')\, \not{e} (\not{p} - \not{k}' + m) \not{e}'\, u(p,s)$

$$= \underline{-\bar{u}(p',s')\, \not{e} \not{k}' \not{e}'\, u(p,s)} - \bar{u}(p',s')\, \not{e}\not{e}' \underbrace{(\not{p} - m) u(p,s)}_{=0}$$

e forts) Vi kan også skrive

$$q^2 - m^2 = (p+k)^2 - m^2 = 2pk + \underbrace{k^2}_{=0} + \underbrace{p^2 - m^2}_{=0} = 2pk$$

og

$$q'^2 - m^2 = (p-k')^2 - m^2 = -2pk' + \underbrace{k'^2}_{=0} + \underbrace{p^2 - m^2}_{=0} = -2pk'$$

Vi får derfor

$$\mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)} = \frac{e^2}{2}\left[\frac{\bar{u}(p',s')\,\not{\epsilon}'\not{k}\,\not{\epsilon}\,u(p,s)}{(pk)} + \frac{\bar{u}(p',s')\,\not{\epsilon}\,\not{k}'\not{\epsilon}'\,u(p,s)}{(pk')}\right]$$

f) Vi har $|\mathcal{M}_{fi}|^2 \propto e^4 \propto \alpha^2$.

Når $\omega \to 0$ er det bare elektronmassen m som kan brukes til å konstruere en lengde-dimensjon,

$$\lambda_e = \frac{\hbar}{mc}$$

Så vi må ha

$$\sigma_{tot} \simeq (\alpha \lambda_e)^2 = r_e^2 =$$

r_e kalles den klassiske elektronradien.

[Det vinkelige tverrsnittet i denne grensen kalles Thomson tverrsnittet, og er

$$\sigma_{Th} = \frac{8\pi}{3} r_e^2$$

Så overslagsregningen gir ca. en faktor 8 for lite.]

$7.92 \cdot 10^{-30}\ m^2$

g)

$$\overline{|\mathcal{M}_{fi}|^2} = \overline{|\mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)}|^2}$$

$$= \overline{|\mathcal{M}_{fi}^{(a)}|^2} + \overline{|\mathcal{M}_{fi}^{(b)}|^2} + \overline{\mathcal{M}_{fi}^{(a)} \mathcal{M}_{fi}^{(b)*}}$$

$$+ \overline{\mathcal{M}_{fi}^{(b)*} \mathcal{M}_{fi}^{(a)*}}.$$

Oppgave 2 forts.

g) $$\mathcal{M}_{fi} = \frac{e^2}{2}\left[\frac{\bar u' \not{e}' \not{k} \not{e} u}{(pk)} + \frac{\bar u' \not{e} \not{k}' \not{e}' u}{(pk')}\right] \Rightarrow$$

$$\overline{|\mathcal{M}_{fi}|^2} = \frac{e^4}{4}\cdot\frac{1}{2}\Big\{ \frac{Tr\{\not{e}' \not{k} \not{e} (\not{p}+m_e) \not{e} \not{k} \not{e}' (\not{p}'+m_e)\}}{(pk)^2}$$

$$+ \frac{Tr[\not{e} \not{k}' \not{e}' (\not{p}+m_e) \not{e}' \not{k}' \not{e} (\not{p}'+m_e)]}{(pk')^2}$$

$$+ \frac{Tr[\not{e}' \not{k} \not{e} (\not{p}+m_e) \not{e} \not{k}' \not{e} (\not{p}'+m_e)]}{(pk)(pk')}$$

$$+ \frac{Tr[\not{e} \not{k}' \not{e}' (\not{p}+m_e) \not{e} \not{k} \not{e}' (\not{p}'+m_e)]}{(pk)(pk')}\Big\}$$

$$= \frac{e^4}{8}\Big\{ \frac{T^{aa}}{(pk)^2} + \frac{T^{bb}}{(pk')^2} + \frac{T^{ab}+T^{ba}}{(pk)(pk')}\Big\}$$

h) $$T^{aa} = Tr\{\not{e}' \not{k} \not{e} \not{p} \not{e} \not{k} \not{e}' \not{p}'\}$$

$$= -Tr\{\not{e}' \not{k} \not{p} \not{e} \not{e} \not{k} \not{e}' \not{p}'\} \qquad \not{e}\not{p} = -\not{p}\not{e} \text{ because } (pe)=0$$

$$= Tr\{\not{e}' \underbrace{\not{k} \not{p}}_{-\not{p}\not{k} + 2(pk)} \not{k} \not{e}' \not{p}'\} \qquad \not{e}\not{e} = e^2 = -1.$$

$$= -Tr\{\not{e}' \not{p} \underbrace{\not{k} \not{k}}_{= k^2 = 0} \not{e}' \not{p}'\} + 2(pk) Tr\{\not{e}' \not{k} \not{e}' \not{p}'\}$$

$$= 2(pk) * 4 [2(ke')(p'e') - \underbrace{(e'e')}_{=-1}(p'k)]$$

$$= 8(pk)[(p'k) + 2(p'e')(ke')]$$

$$= 8(pk)(p'k)$$ [redacted] $$16(ke')^2 \qquad p'e' = \underbrace{pe'}_{=0} + \underbrace{k'e'}_{=0} - ke'$$