NTNU **Institutt for fysikk**

Faglig kontakt under eksamen:
Professor Kåre Olaussen
Telefon: 9 36 52 eller 45 43 71 70

Eksamen i FY3466 KVANTEFELTTEORI II
Tirsdag 20. mai 2008
09:00–13:00

Tillatte hjelpemidler: Alternativ **C**

Typegodkjent kalkulator, med tomt minne (i henhold til NTNU's liste).
K. Rottman: *Matematisk formelsamling* (alle språkutgaver).
Schaum's Outline Series: *Mathematical Handbook of Formulas and Tables.*

There is an english version of this exam at the end.

Dette oppgavesettet er på 3 sider.

Oppgave 1. Modell for fermioner og bosoner

I denne oppgaven skal du se på en sterkt forenklet versjon av Glashow-Weinberg-Salam (GWS) modellen for elektro-svake vekselvirkninger. Se først på Lagrangetettheten

$$\mathcal{L} = \mathrm{i}L^\dagger \bar{\sigma}^\nu\,\partial_\nu L + \mathrm{i}R^\dagger \sigma^\nu\,\partial_\nu R + (\partial^\nu\varphi^*)\,(\partial_\nu\varphi) + \mu^2\varphi^*\varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2, \tag{1}$$

der μ^2 og λ er positive parametre. I ligning (1) er L og R to-komponent spinorfelter, $\sigma^\nu = (I, \boldsymbol{\sigma})$ og $\bar{\sigma}^\nu = (I, -\boldsymbol{\sigma})$ der $\boldsymbol{\sigma}$ er Pauli-matrisene.

a) Hva er den klassiske grunntilstanden (minimum energi tilstanden) for denne modellen? Dvs, hvilke verdier har feltene L, R og φ i denne tilstanden, og hva er den tilhørende energitettheten?

b) Hva er, til laveste orden i parameteren λ, massene til partiklene (de kvantiserte fluktuasjonene om grunntilstanden) i denne modellen?

c) Denne modellen er invariant under tre uavhengige globale fasetransformasjoner, $U(1) \times U(1) \times U(1)$? Hvilke fasetransformasjoner er det?

Vi adderer nå et vekselvirkningsledd til Lagrangetettheten,

$$\Delta\mathcal{L} = -\lambda_m \left(\varphi\, L^\dagger R + \varphi^* R^\dagger L\right) \tag{2}$$

d) Denne modellen blir nå invariant under *to* uavhengige globale fasetranstransformasjoner, $U(1) \times U(1)$. Vis hvordan disse to transformasjonene virker på de tre feltene.

e) Hva blir massen til fermionet (til laveste orden i λ og λ_m) i den modifiserte modellen?

f) De to globale fasetransformasjonene kan gjøres lokale ved å innføre to gaugefelter, B_ν og W_ν, med tilhørende kovariant deriverte. Finn et konsistent sett med kovariant deriverte $(D_\nu^{(L)}, D_\nu^{(R)}, D_\nu^{(\varphi)})$, og skriv ned den nye Lagrangetettheten for modellen.

Du kan anta at de kovariant deriverte ikke innholder vekselvirkningsparametre, men at de kinetiske leddene for gaugefeltene er

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4g^2} W_{\nu\sigma} W^{\nu\sigma} - \frac{1}{4g'^2} B_{\nu\sigma} B^{\nu\sigma}, \tag{3}$$

der $B_{\nu\sigma} = \partial_\nu B_\sigma - \partial_\sigma B_\nu$ og $W_{\nu\sigma} = \partial_\nu W_\sigma - \partial_\sigma W_\nu$.

g) En lineærkombinasjon av gaugefeltene B_ν og W_ν får kvanter med masse M_Z i denne modellen. Hvilken lineærkombinasjon blir det? Finn M_Z^2 uttrykt ved parametrene μ^2, λ, g og g'.

Oppgave 2. Begreper i kvantefeltteori

Gi en kort kvalitativ beskrivelse av følgende begreper

a) Ward identitet.

b) Dimensjonell regularisering.

c) Wick rotasjon (i Feynman diagram integrasjoner).

d) Cabibbo-Kobayashi-Maskawa matrisen.

e) Nøytrino oscillasjoner.

f) Spontant symmetribrudd.

g) Goldstone boson.

h) Landau, Feynman og Yennie gauge.

Oppgave 3. Grassman integrasjon

La $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ være en generell kompleks 2×2 matrise, θ_1, θ_2, θ_1^*, θ_2^* uavhengige Grassmann variable, og

$$S_1 = (\theta_1, \theta_2)\, \boldsymbol{A} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \tag{4}$$

$$S_2 = (\theta_1^*, \theta_2^*)\, \boldsymbol{A} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}. \tag{5}$$

Utfør følgende Grassmann integraler

a)

$$I_1 = \int \mathrm{d}\theta_1 \mathrm{d}\theta_2\, \mathrm{e}^{S_1}. \tag{6}$$

b)

$$I_2 = \int \mathrm{d}\theta_1 \mathrm{d}\theta_2\, \theta_1 \theta_2\, \mathrm{e}^{S_1}. \tag{7}$$

c)

$$I_3 = \int \mathrm{d}\theta_1^*\, \mathrm{d}\theta_2^*\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2\, \mathrm{e}^{S_2}. \tag{8}$$

d)

$$I_4 = \int \mathrm{d}\theta_1^*\, \mathrm{d}\theta_2^*\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2\, \theta_1^*\, \theta_2\, \mathrm{e}^{S_2}. \tag{9}$$

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Exam in FY3466 QUANTUM FIELD THEORY II

Tuesday May 20, 2008
09:00–13:00

Allowed help: Alternativ **C**

Standard pocket calculator (according to list made by NTNU).
K. Rottman: *Matematisk formelsamling* (any language).
Schaum's Outline Series: *Mathematical Handbook of Formulas and Tables.*

This problem set consists of 3 pages.

Problem 1. Model for fermions and bosons

In this problem we shall investigate a much simplified version of the Glashow-Weinberg-Salam (GWS) model of electroweak interactions. Consider first the Lagrangian density

$$\mathcal{L} = \mathrm{i}L^\dagger \bar{\sigma}^\nu\, \partial_\nu L + \mathrm{i}R^\dagger \sigma^\nu\, \partial_\nu R + \left(\partial^\nu \varphi^*\right)\left(\partial_\nu \varphi\right) + \mu^2 \varphi^* \varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2, \tag{1}$$

where μ^2 and λ are positive parameters. In equation (1) L and R are two-component spinor fields, $\sigma^\nu = (I, \boldsymbol{\sigma})$ and $\bar{\sigma}^\nu = (I, -\boldsymbol{\sigma})$ where $\boldsymbol{\sigma}$ are the Pauli matrices.

a) What is the classical groundstate (minimum energy state) of this model? I.e., what are the values of the fields L, R and φ in this state, and what is the corresponding energy density?

b) What are, to lowest order in the parameter λ, the masses of the particles (the quantized fluctuations around the ground state) in this model?

c) This model is invariant under three independent global phase transformations, $U(1) \times U(1) \times U(1)$? Which phase transformations?

We now add an interaction term to the Lagrangian density,

$$\Delta\mathcal{L} = -\lambda_m \left(\varphi\, L^\dagger R + \varphi^* R^\dagger L\right) \tag{2}$$

d) This model now becomes invariant under *two* independent global phase transformations, $U(1) \times U(1)$. Show how these two transformations act on the three fields.

e) What is the mass of the fermion (to lowest order in λ and λ_m) in the modified model?

f) The two global phase transformations can be made local by introducing two gaugefields, B_ν og W_ν, and the associated covariant derivatives. Find a consistent set of covariant derivatives ($D_\nu^{(L)}$, $D_\nu^{(R)}$, $D_\nu^{(\varphi)}$), and write down the new Lagrangian density for the model.

You may assume that the covariant derivatives don't contain any interaction parameters, but that the kinetic terms for the gauge fiels are

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4g^2} W_{\nu\sigma} W^{\nu\sigma} - \frac{1}{4g'^2} B_{\nu\sigma} B^{\nu\sigma}, \tag{3}$$

where $B_{\nu\sigma} = \partial_\nu B_\sigma - \partial_\sigma B_\nu$ and $W_{\nu\sigma} = \partial_\nu W_\sigma - \partial_\sigma W_\nu$.

g) A linear combination of the gauge fields B_ν and W_ν gets quanta with mass M_Z in this model. Which linear combination? Find M_Z^2 expressed by the parameters μ^2, λ, g and g'.

Problem 2. Concepts in Quantum Field Theory

Give a short qualitative description of the following concepts

a) Ward identity.

b) Dimensional regularization.

c) Wick rotation (in Feynman diagram integrations)

d) The Cabibbo-Kobayashi-Maskawa matrix.

e) Neutrino oscillations.

f) Spontaneous symmetry breakdown.

g) Goldstone boson.

h) Landau, Feynman, and Yennie gauge.

Problem 3. Grassman integration

Let $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a general complex 2×2 matrix, θ_1, θ_2, θ_1^*, θ_2^* independent Grassmann variables, and

$$S_1 = (\theta_1, \theta_2)\, \boldsymbol{A} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \tag{4}$$

$$S_2 = (\theta_1^*, \theta_2^*)\, \boldsymbol{A} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}. \tag{5}$$

Do the following Grassmann integrals

a)

$$I_1 = \int \mathrm{d}\theta_1 \mathrm{d}\theta_2 \, \mathrm{e}^{S_1}. \tag{6}$$

b)

$$I_2 = \int \mathrm{d}\theta_1 \mathrm{d}\theta_2 \, \theta_1 \theta_2 \, \mathrm{e}^{S_1}. \tag{7}$$

c)

$$I_3 = \int \mathrm{d}\theta_1^* \, \mathrm{d}\theta_2^* \, \mathrm{d}\theta_1 \, \mathrm{d}\theta_2 \; \mathrm{e}^{S_2}. \tag{8}$$

d)

$$I_4 = \int \mathrm{d}\theta_1^* \, \mathrm{d}\theta_2^* \, \mathrm{d}\theta_1 \, \mathrm{d}\theta_2 \; \theta_1^* \, \theta_2 \, \mathrm{e}^{S_2}. \tag{9}$$