NTNU Trondheim, Institutt for fysikk

Home exam FY3466 Advanced Quantum Field Theory

The process $\mu \to e + \gamma$.
In the SM with non-zero Dirac masses for neutrinos the lepton flavor numbers L_i ($i = e, \mu, \tau$) are not conserved. Therefore the $W\bar{l}_i\nu_j$ vertex contains a mixing matrix element U^*_{ij} and and processes like $\mu \to e + \gamma$ are allowed.
a.) Write the amplitude $\mathcal{A}(\mu \to e + \gamma)$ as $\mathcal{A}(\mu \to e + \gamma) = \varepsilon_\lambda \langle e| J^\lambda_{\rm em} |\mu\rangle$ and decompose it in Lorentz invariant functions

$$\langle e| J^\lambda_{\rm em} |\mu\rangle = \bar{u}_e(p-q)[A(q^2)\gamma^\lambda + \ldots]u_\mu(p)\,.$$

Use current conservation, $\partial_\lambda J^\lambda_{\rm em} = 0$, and the on-shell condition $q^2 = 0$ to restrict these functions. Apply the Gordon decomposition to show that you have to calculate only terms $p \cdot \varepsilon$ (analogous to the calculation of the anamalous magnetic moment, see Zee or ch. 8.1 of the lecture notes). You can set $m_e = 0$ throughout; you have to keep only the leading order in the neutrino masses m_i.
b.) Draw all 1-loop Feynman diagrams in R_ξ consistent with the external particles; which one(s) you have to calculate in unitary gauge, which one(s) for a general R_ξ gauge?

Decide if you do part c.), d.) or (for the dedicated student) e.)

c.) Calculate the Feynman diagram(s) relevant in unitary gauge using the Feynman rules for general R_ξ gauge (i.e. keeping $\xi \neq 1$ as a free parameter). Perform then the limit $\xi \to \infty$. Square and calculate the decay width.
d.) Calculate the relevant Feynman diagram(s) in R_ξ gauge using $\xi = 1$. Square and calculate the decay width.
e.) Calculate the relevant Feynman diagram(s) in general R_ξ gauge and verify that the sum is independent of ξ.

Two hints: There is the leptonic version of the GIM mechanism at work, when you sum over the three generations in the intermediate state: Thus the term leading in m_i is obtained as

$$\sum_i \frac{U^*_{ei}U_{\mu i}}{(p+k)^2 - m_i^2} = \sum_i U^*_{ei}U_{\mu i}\left[\frac{1}{(p+k)^2} + \frac{m_i^2}{(p+k)^4} + \ldots\right] = \sum_i U^*_{ei}U_{\mu i}\frac{m_i^2}{(p+k)^4} + \ldots$$

For c.) or e.): It is useful to split the gauge boson propagator into

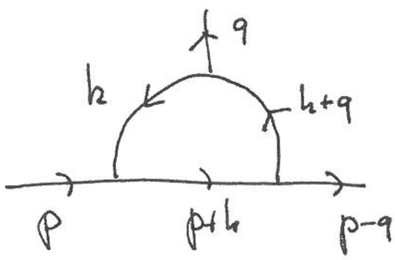
$$D_F^{\mu\nu}(k^2) = \frac{-(\eta^{\mu\nu} - k^\mu k^\nu/M^2)}{k^2 - M^2 + {\rm i}\varepsilon} \tag{1}$$

$$+ \frac{-k^\mu k^\nu/M^2}{k^2 - \xi M^2 + {\rm i}\varepsilon}\,. \tag{2}$$

The structure of the $WW\gamma$ vertex implies that one combination gives zero contribution.

For a complete set of Feynman rules see e.g. Jorge C. Romao and Joao P. Silva, arXiv:1209.6213 [hep-ph], the relevant ones are attached. In both cases, you have to add leptonic mixing matrices.

Please assign momenta as



for an easier comparison of results.

The FP ghost-field Lagrangian (9.69) is then given by

$$\mathscr{L}_{\mathrm{FPG}} = \int \mathrm{d}^4x\,\mathrm{d}^4y(\omega_i^\dagger(x), \chi^\dagger(x)) \begin{pmatrix} M_f(x,y)_{ij} & M_f(x,y)_i \\ M_f(x,y)_j & M_f(x,y) \end{pmatrix} \begin{pmatrix} \omega_j(y) \\ \chi(y) \end{pmatrix}. \tag{B.48}$$

Propagators and vertices for bosons and FP ghosts

Define the physical vector bosons as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp \mathrm{i}A_\mu^2)$$

$$Z_\mu = \cos\theta_{\mathrm{W}} A_\mu^3 - \sin\theta_{\mathrm{W}} B_\mu$$

$$A_\mu = \sin\theta_{\mathrm{W}} A_\mu^3 + \cos\theta_{\mathrm{W}} B_\mu$$

and write the scalar mesons as

$$\phi' = \begin{pmatrix} \phi^+ \\ \dfrac{\phi_1 + \mathrm{i}\phi_2}{\sqrt{2}} \end{pmatrix}.$$

For the ghost fields we can define similar combinations

$$\omega^\pm = \frac{1}{\sqrt{2}}(\omega_1 \mp \mathrm{i}\omega_2)$$

$$\omega_Z = \cos\theta_{\mathrm{W}}\omega_3 - \sin\theta_{\mathrm{W}}\chi$$

$$\omega_\gamma = \sin\theta_{\mathrm{W}}\omega_3 + \cos\theta_{\mathrm{W}}\chi.$$

With these definitions, we can easily work out the propagators from the quadratic part of the Lagrangian $\mathscr{L}_1 + \mathscr{L}_3 + \mathscr{L}_{\mathrm{gf}} + \mathscr{L}_{\mathrm{FPG}}$ in eqns (11.37), (11.59), (B.41), and (B.48)

$\mathrm{W}^\pm$ $\quad \dfrac{-\mathrm{i}}{k^2 - M_{\mathrm{W}}^2 + \mathrm{i}\varepsilon}[g_{\mu\nu} + (\xi - 1)k_\mu k_\nu/(k^2 - \xi M_{\mathrm{W}}^2)]$

Z $\quad \dfrac{-\mathrm{i}}{k^2 - M_{\mathrm{Z}}^2 + \mathrm{i}\varepsilon}[g_{\mu\nu} + (\xi - 1)k_\mu k_\nu/(k^2 - \xi M_{\mathrm{Z}}^2)]$

$\phi^\pm$ $\quad \dfrac{\mathrm{i}}{k^2 - \xi M_{\mathrm{W}}^2 + \mathrm{i}\varepsilon}$

ϕ_2 $\quad \dfrac{\mathrm{i}}{k^2 - \xi M_{\mathrm{Z}}^2 + \mathrm{i}\varepsilon}$

ϕ_1 $\quad \dfrac{\mathrm{i}}{k^2 - 2\mu^2 + \mathrm{i}\varepsilon}$

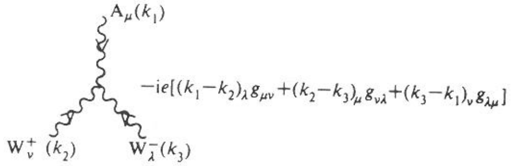
$\omega^\pm$ $\quad \dfrac{-\mathrm{i}}{k^2 - \xi M_{\mathrm{W}}^2 + \mathrm{i}\varepsilon}$

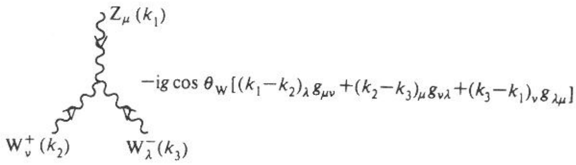
ω_{Z} $\quad \dfrac{-\mathrm{i}}{k^2 - \xi M_{\mathrm{Z}}^2 + \mathrm{i}\varepsilon}$

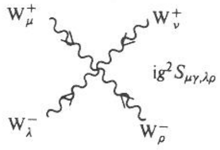
ω_γ $\quad \dfrac{-\mathrm{i}}{k^2 + \mathrm{i}\varepsilon}$

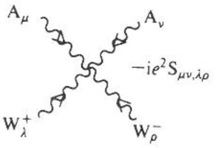
where $\xi = 1$ 't Hooft–Feynman gauge, $\xi = 0$ Landau gauge, and $\xi = \infty$ unitary gauge.

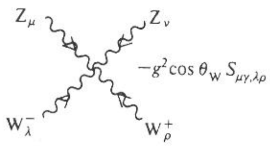
The boson vertices are



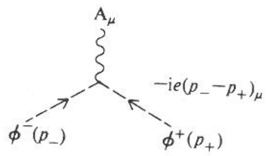


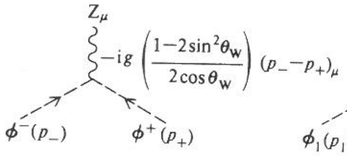


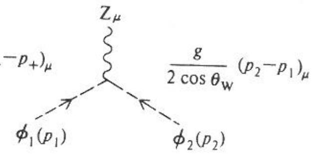


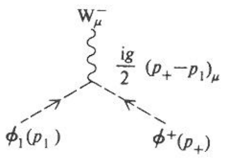


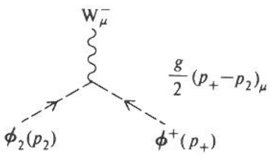
with $S_{\mu\nu,\lambda\rho} = 2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}$. In graphs below *all* charged boson lines are taken to be entering *into* the vertices.

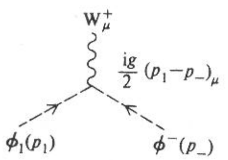


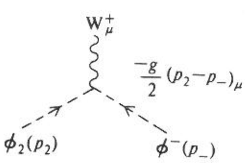


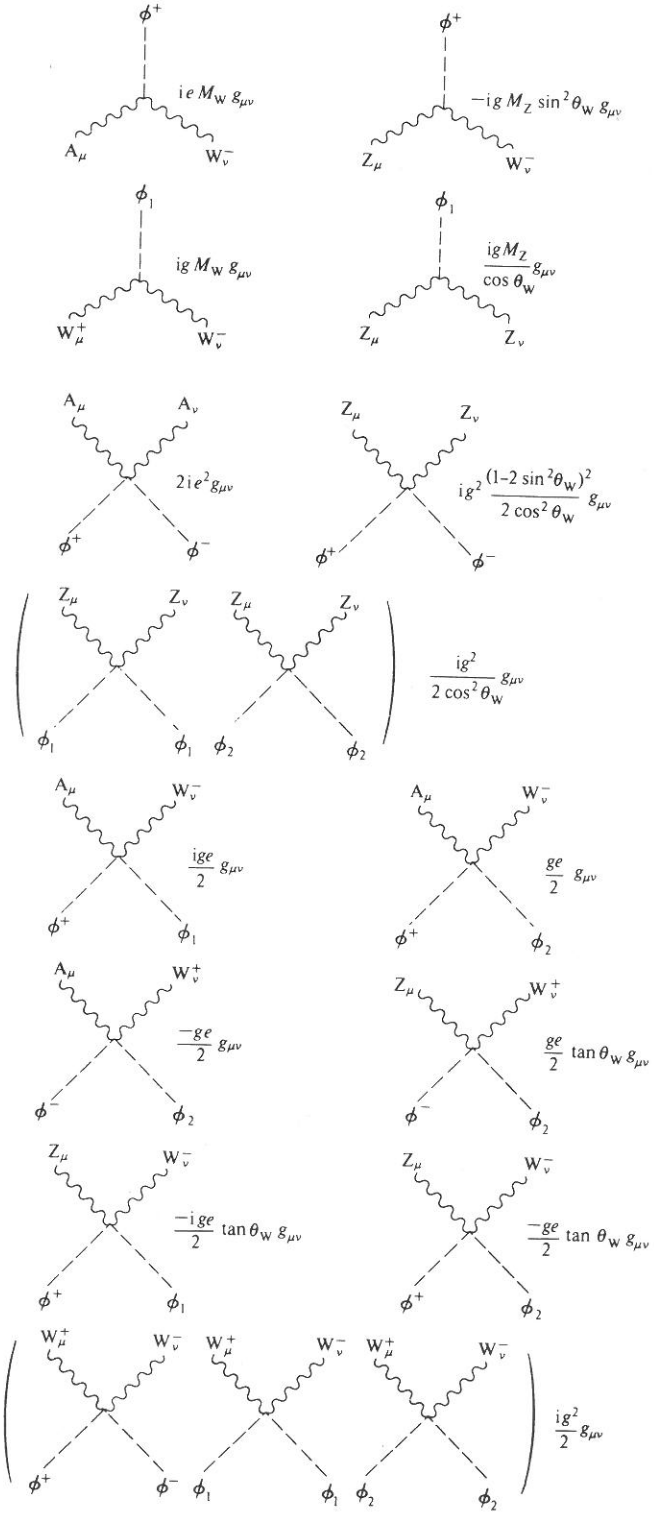











ϕ^+
$ieM_W g_{\mu\nu}$
A_μ W_ν^-
ϕ^+
$-igM_Z \sin^2\theta_W g_{\mu\nu}$
Z_μ W_ν^-
ϕ_1
$igM_W g_{\mu\nu}$
W_μ^+ W_ν^-
ϕ_1
$\frac{igM_Z}{\cos\theta_W} g_{\mu\nu}$
Z_μ Z_ν
A_μ A_ν
$2ie^2 g_{\mu\nu}$
ϕ^+ ϕ^-
Z_μ Z_ν
$ig^2 \frac{(1-2\sin^2\theta_W)^2}{2\cos^2\theta_W} g_{\mu\nu}$
ϕ^+ ϕ^-
Z_μ Z_ν Z_μ Z_ν
ϕ_1 ϕ_1 ϕ_2 ϕ_2
$\frac{ig^2}{2\cos^2\theta_W} g_{\mu\nu}$
A_μ W_ν^-
$\frac{ige}{2} g_{\mu\nu}$
ϕ^+ ϕ_1
A_μ W_ν^-
$\frac{ge}{2} g_{\mu\nu}$
ϕ^+ ϕ_2
A_μ W_ν^+
$\frac{-ge}{2} g_{\mu\nu}$
ϕ^- ϕ_2
Z_μ W_ν^+
$\frac{ge}{2} \tan\theta_W g_{\mu\nu}$
ϕ^- ϕ_2
Z_μ W_ν^-
$\frac{-ige}{2} \tan\theta_W g_{\mu\nu}$
ϕ^+ ϕ_1
Z_μ W_ν^-
$\frac{-ge}{2} \tan\theta_W g_{\mu\nu}$
ϕ^+ ϕ_2
W_μ^+ W_ν^- W_μ^+ W_ν^- W_μ^+ W_ν^-
ϕ^+ ϕ^- ϕ_1 ϕ_1 ϕ_2 ϕ_2
$\frac{ig^2}{2} g_{\mu\nu}$

Inclusion of leptons and quarks

Propagator: $\xrightarrow{\;p\;}$ $\qquad \dfrac{\mathrm{i}}{\not p - m_i + \mathrm{i}\varepsilon}.$

Vertices for leptons: $l = (\mathrm{e}, \mu, \tau)$, $\nu_l = (\nu_e, \nu_\mu, \nu_\tau)$

for quarks q: p = (u, c, t), n = (d, s, b) with the CKM mixing matrix U_{pn} of eqn (12.39).