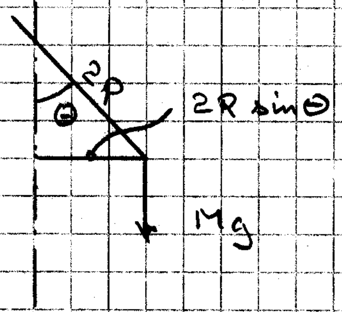
SIF 4002

27. mai 1998

Løsningsskisse

Oppgave 1

a



Dreiemoment motsatt Θ-retning;

$\tau = -Mg \cdot 2R \sin\Theta \xrightarrow[\Theta \text{ liten}]{} Mg \cdot 2R\Theta$

$\tau = \bar{I}\ddot{\Theta}$ med $\bar{I} = \frac{9}{2}MR^2$

$\Rightarrow \frac{9}{2}MR^2\ddot{\Theta} + Mg \cdot 2R\Theta = 0$

$\Rightarrow \boxed{\ddot{\Theta} + \frac{4}{9}\frac{g}{R}\Theta = 0}$

Dette er ligm. for enkel harmonisk bevegelse, type $\ddot{x} + \omega_0^2 x = 0$

med $\omega_0 = \sqrt{4g/9R} = (2/3)\sqrt{g/R}$

b. $t_0 = 2\pi/\omega_0 \Rightarrow \omega_0^2 = \frac{4}{9}\frac{g}{R} = \left(\frac{2\pi}{t_0}\right)^2 \Rightarrow R = \frac{4}{9}g\left(\frac{t_0}{2\pi}\right)^2$

$\underline{R} = (g/9)(t_0/\pi)^2 = \underline{27{,}6 \text{ mm}}$

$t_0 \propto \sqrt{R} \Rightarrow \Delta t_0/t_0 = \frac{1}{2}\Delta R/R = \frac{1}{2}\alpha\Delta T$

$\underline{\alpha} = 2(\Delta t_0/t_0)/\Delta T = 2\cdot(7\text{s}/24\cdot 60\cdot 60\text{s})/8\text{K} = \underline{2{,}0\cdot 10^{-5}\ \text{K}^{-1}}$

c Treghetsmomentet om tyngdepunktet er

$I_T = \sum m_i r_i^2 \rightarrow \int dm \cdot r^2$

$dm = (M/\pi R^2)\cdot d(\pi r^2) = 2Mr\,dr/R^2$

$I_T = (2M/R^2)\int_0^R r^3\,dr = (2M/R^2)(R^4/4) = \frac{1}{2}MR^2$

$\underline{\bar{I}} = I_T + M\cdot(2R)^2 = (\frac{1}{2}+4)MR^2 = \underline{\frac{9}{2}MR^2}$

Oppgave 2

a. $\underline{V} = nRT_a/p_a = (0.01 \cdot 8.3145 \cdot 300/10^5)\ m^3 = \underline{\underline{249.4\ cm^3}}$

(som i en 250 cc motorsykkel)

Oppvarming ved konst. volum: $C_V = \nu \cdot \frac{1}{2}R = \frac{5}{2}R$

$\underline{Q} = nC_V \Delta T = n \cdot \frac{5}{2}R \cdot (T_d - T_a) = \underline{\underline{20.8\ J}}$

b. $T_b V_b^{\gamma-1} = T_a V_a^{\gamma-1} \Rightarrow \underline{T_b} = T_a (V_a/V_b)^{\gamma-1} = T_a r^{\gamma-1} = 300 \cdot 9^{0.4} = \underline{\underline{722.5\ K}}$

$p_b V_b^{\gamma} = p_a V_a^{\gamma} \Rightarrow \underline{p_b} = p_a r^{\gamma} = 10^5 \cdot 9^{1.4} = \underline{\underline{2.17\ MPa}}$

$dQ = 0 = dU + p\,dV \Rightarrow p\,dV = -dU$

Arbeid utført på gassen er

$\underline{W_{ab}} = -\int_a^b p\,dV = U_b - U_a = nC_V(T_b - T_a)$

$= 0.01 \cdot 20.8 \cdot (722.5 - 300)\ J = \underline{\underline{87.9\ J}}$

c. $dQ = 0 = dU + p\,dV$ (adiabatisk)

$dU = nC_V\,dT$ og $p = nRT/V$ $\Rightarrow$

$nC_V\,dT + nRT\,dV/V = 0 \quad : \times(1/nC_VT) \Rightarrow$

$dT/T + (R/C_V)\,dV/V = 0$

$d(\ln T + (R/C_V)\ln V) = d\ln(TV^{R/C_V}) = 0 \Rightarrow$

$\underline{\underline{TV^{R/C_V} = \text{konst}}}$

$R/C_V = \gamma - 1$:

$nC_p = (dQ/dT)_{p=\text{konst}} = [(dU + p\,dV)/dT]_{p=\text{konst}}$

$= nC_V + p\,(dV/dT)_{p=\text{konst}}$

$V = nRT/p \Rightarrow (dV/dT)_{p=\text{konst}} = nR/p \Rightarrow$

$C_p = C_V + R \Rightarrow R = C_p - C_V \Rightarrow \underline{R/C_V} = C_p/C_V - 1 = \underline{\gamma - 1}$

Dermed: $\boxed{TV^{\gamma-1} = \text{konst}}$

Oppgave 3

a. $L = \mu_0 N^2 A/l \Rightarrow \underline{N} = \sqrt{Ll/\mu_0 A} = \underline{\underline{281}}$

Lengde av viklingene $s = N \cdot \pi D = 7.06\,\text{m}$

$\underline{R} = s \cdot 0.54\,\Omega/\text{m} = \underline{\underline{3.8\,\Omega}}$

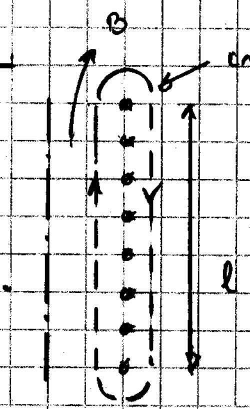
$\mu_r = 4000 \Rightarrow \underline{L} \to \mu_r \cdot L = \underline{\underline{200\,\text{mH}}}$

b. Spenningsbalanse $v_R + v_L = V_s \Rightarrow Ri + L\,di/dt = V_s$

$$\boxed{(L/R)\,di/dt + i = V_s/R}$$

$I = I_0 (1 - e^{-t/\tau})$; $\underline{I_0} = V_s/R = \underline{\underline{1.5\,\text{A}}}$; $\underline{\tau} = L/R = \underline{\underline{50\,\text{ms}}}$

c.



$B \approx$ konstant (og langs aksen)

inne i spolen, og null utenfor

$\Rightarrow \oint \vec{B} \cdot d\vec{s} \approx B \cdot l$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 \sum i = \mu_0 N \cdot i$

$\Rightarrow \underline{B = \mu_0 N i/l}$

$\underline{\Phi_B = \mu_0 N A i/l}$

Spenning indusert i én vikling: $v^{(1)} = -\epsilon^{(1)} = d\Phi_B/dt$

Spenning indusert i N viklinger:

$\underline{v} = N v^{(1)} = N\,d\Phi_B/dt = \underline{(\mu_0 N^2 A/l)\,di/dt}$

Sammenligning med def. ligning for L: $v = L\,di/dt$

$\Rightarrow \boxed{L = \mu_0 N^2 A/l}$