SIF 4002 Fysikk

Torsdag 12. august 1999

Løsningsskisse

Mangler ved oppgavene:

1. Oppg. 2: konstanten i svingeligningen; d^2 skal være d^4
2. Oppg. 3: Tallverdien for λ_{is} ikke oppgitt
3. Oppg. 4: Enheten for viskositet er $[\mathrm{Pa \cdot s}] = (\mathrm{N/m^2})\cdot \mathrm{s}$ (uklart skrevet)

Oppg. 1a

Energi ved hoppkanten: $W_{kin} + V_{pot} \Rightarrow W_{kin} = mgh$

$W_{kin} = W_{trans} + W_{rot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$I = (2/5) m r^2$ og $\omega = v/r \Rightarrow W_{kin} = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot v^2/r^2 = \frac{7}{10} m v^2$

$(7/10) m v^2 = mgh \Rightarrow v^2 = (10/7) g h$ og $\underline{\underline{v = \sqrt{10 g h / 7}}}$

Svevet: $x(t) = vt$; $y(t) = -\frac{1}{2} g t^2$

Nedslag $t = t_0$: $y = -kx$): $-\frac{1}{2} g t_0^2 = -k v t_0 \Rightarrow \underline{\underline{t_0 = 2 k v / g}}$

Hopplengde: $s^2 = x^2 + y^2 = v^2 t_0^2 + \frac{1}{4} g^2 t_0^4$

$= v^2 t_0^2 (1 + k^2) = 4 (v^4/g^2) k^2 (1 + k^2)$; $v^2/g = 10/7\, h \Rightarrow$

$\underline{\underline{s = (20/7)\, h\, k \sqrt{1 + k^2}}} \Rightarrow \underline{s} = (5\sqrt{5}/7)\,\mathrm{m} = \underline{\underline{1{,}597\,\mathrm{m}}}$

Oppg. 1b

$F_{\parallel} = mg \sin\theta$; $F_{\perp} = mg \cos\theta$

$$\left.\begin{aligned} a &= dv/dt = (F_{\parallel} - F_s)/m \\ \alpha &= d\omega/dt = \tfrac{1}{r}\, dv/dt = F_s \cdot r / I = F_s \cdot r / \tfrac{2}{5} m r^2 \Rightarrow dv/dt = \tfrac{5}{2} F_s / m \end{aligned}\right\} \Rightarrow$$

$(F_{\parallel} - F_s)/m = \frac{5}{2} F_s / m \Rightarrow \underline{F_s} = (2/3) F_{\parallel} = \underline{(2/3)\, mg \sin\theta}$

$F_s^{max} = \mu_s F_{\perp} = \mu_s mg \cos\theta \Rightarrow \frac{2}{3} mg \sin\theta_{max} = \mu_s mg \cos\theta_{max}$

$\underline{\operatorname{tg}\theta_{max} = \frac{3}{2}\mu_s}$ $\quad(\theta_{max} = \operatorname{arctg}(\frac{3}{2}\mu_s))$

(lengde – ikke akse)

$\operatorname{tg}\theta = dy/dx = -2x/a \Rightarrow x^{max} = \frac{1}{2} a \operatorname{tg}\theta_{max} = -\frac{3}{4}\mu_s a$

$\underline{h_{max}} = y(x_{max}) = \underline{\underline{\frac{9}{16}\mu_s^2 a}}$

Oppg. 2a

Statisk forlengelse y_0 : $mg = ky_0 \Rightarrow \underline{y_0 = mg/k}$

Krefter på m:

$$F = -k(y+y_0) - b\dot{y} - mg = -ky - b\dot{y}$$

$F = m\ddot{y} \Rightarrow$

$$\boxed{\ddot{y} + (b/m)\dot{y} + (k/m)y = 0}$$

$\{\ddot{y} + 2\delta\dot{y} + \omega_0^2 y = 0 \;;\; \delta = \frac{1}{2}b/m$ og $\omega_0 = \sqrt{k/m}\}$

$y(t) = Ae^{-\delta t}\cos(\omega_d t + \phi) \;;\; \omega_d^2 = \omega_0^2 - \delta^2$

$$\left.\begin{aligned} y(0) &= y_0 = A\cos\phi \\ \dot{y}(0) &= 0 = -\delta A\cos\phi - \omega_d A\sin\phi \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} \phi &= -\arctg(\delta/\omega_d) \\ A &= y_0\sqrt{1-(\delta/\omega_0)^2} \end{aligned}}$$

Tallverdier: $y_0 = 0{,}098\,m$; $\omega_0 = 10\,s^{-1}$; $\delta = 2\,s^{-1}$

$\Rightarrow$ $\underline{\underline{\phi = -0{,}201\text{ rad}}} = \underline{\underline{-11{,}5^\circ}}$; $\underline{\underline{A = 0{,}096\,m}}$

Oppg. 2b.

$\Delta V(y) = y \cdot A$ hvor $A = \pi r^2 = \pi d^2/4$

$pV^\gamma = \text{konst.} \Rightarrow \Delta p \cdot V^\gamma + p \cdot \Delta(V^\gamma) = 0 \;;\; \Delta(V^\gamma) = \gamma V^{\gamma-1}\Delta V \Rightarrow$

$$\underline{\Delta p} = -\gamma p \Delta V/V = \underline{-\gamma p y A/V}$$

$F = M\ddot{y}$ med $F = A \cdot \Delta p \Rightarrow$

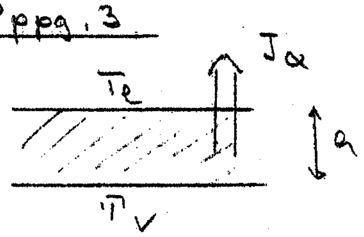
$$\boxed{\ddot{y} + (\gamma A^2 p/MV)y = 0} \qquad \underline{\omega_0^2 = \gamma \cdot \left(\frac{\pi d^2}{4}\right)^2 p/MV}$$

$\tau = 2\pi/\omega_0 = 8\sqrt{MV/\gamma d^4 p}$

Énatomig gass, 3 frihetsgrader (f), $\gamma = (f+2)/f = 5/3 \Rightarrow$

$$\boxed{\tau = 0{,}871\,s}$$

Oppg. 3



$J_Q = -\lambda\, dT/dx \rightarrow -\lambda \dfrac{T_e - T_v}{h} = \lambda \dfrac{T_v - T_e}{h}$

Varmemengde gjennom areal A i tid Δt:

$\Delta Q = J_Q \cdot A \cdot \Delta t = L \cdot \Delta m$ ("frysevarme")

$$\frac{\Delta m}{A\,\Delta t} = \frac{J_Q}{L} = \frac{\lambda}{L}\,\frac{T_v - T_L}{h}$$

Δm: frosset masse i volum $A \cdot \Delta h$; $\Delta m = \rho A \Delta h$ $\Rightarrow$

$$\boxed{\frac{\Delta h}{\Delta t} = \frac{\lambda}{L}\,\frac{T_v - T_L}{h}}$$

(og så manglet tallverdi for λ)

Oppg. 4

Uten viskøst trykkfall ville v vært gitt av: $\rho v^2/2 = \rho g h$

Med ——"—— Δp fåes: $\rho v^2/2 = \rho g h - \Delta p$

$$\boxed{v = \sqrt{2gh - 2\Delta p/\rho}}$$

Vannstrømmen $Q = v \cdot A = v \cdot \pi R^2$ $(R = d/2)$, det vil si:

$v \cdot \pi R^2 = (\pi/8)(R^4/\eta)\,\Delta p / l$

$$\boxed{\Delta p(v) = (8\eta l / R^2)\, v}$$

Innsetting i $v = \sqrt{\cdots}$ gir en 2. gradslign. for v,

$$v^2 + (16\eta l/\rho R^2)\, v - 2gh = 0$$

med løsning

$$v = -8\eta l/\rho R^2 + \sqrt{2gh + (8\eta l/\rho R^2)^2}$$

Tallverdier $\Rightarrow$ $8\eta l/\rho R^2 = 9{,}600$ m/s, $\sqrt{2gh} = 14.00$ m/s

$\Rightarrow$ $\underline{\underline{v = 7.38 \text{ m/s}}}$; $\underline{Q} = 0.58 \cdot 10^{-3}$ m^3/s $= \underline{\underline{0{,}58\ \ell/s}}$