SIF 4003

Oppgave 1

(1)

Gitterkonstant $d = \frac{1\,mm}{500} = 2 \cdot 10^{-6}\,m = 2\,\mu m$

a) Hovedmax: $d \sin\theta = m\lambda$ $\qquad m = 0, \pm 1, \pm 2 \cdots$

$m = 1$:

$$\sin\theta_1 = \frac{\lambda_1}{d} = \frac{589 \cdot 10^{-9}}{2 \cdot 10^{-6}} \Rightarrow \theta_1 = \underline{17{,}13^\circ}$$

$$\sin\theta_2 = \frac{\lambda_2}{d} = \frac{589{,}6 \cdot 10^{-9}}{2 \cdot 10^{-6}} \Rightarrow \theta_2 = \underline{17{,}15^\circ}$$

$m = 3$:

$$\sin\theta_1 = \frac{3\lambda_1}{d} = \frac{3 \cdot 589 \cdot 10^{-9}}{2 \cdot 10^{-6}} \Rightarrow \theta_1 = \underline{62{,}07^\circ}$$

$$\sin\theta_2 = \frac{3 \cdot \lambda_2}{d} = \frac{3 \cdot 589.6 \cdot 10^{-9}}{2 \cdot 10^{-6}} \Rightarrow \theta_2 = \underline{62{,}18^\circ}$$

b)

$$\sin\theta = \frac{m\lambda}{d} \leq 1 \qquad \Rightarrow m \leq \frac{d}{\lambda}$$

$$\frac{d}{\lambda_1} = \frac{2 \cdot 10^{-6}}{589 \cdot 10^{-9}} = 3.40$$

$$\frac{d}{\lambda_2} = \frac{2 \cdot 10^{-6}}{589{,}6 \cdot 10^{-9}} = 3.39$$

For begge bølgelengder er største verdi for $m = 3$
$\Rightarrow$ 7 hovedmaksima som svarer til
$\underline{m = 0, \pm 1, \pm 2, \pm 3}$

c) Oppløsningsevne $R = \frac{\lambda}{\Delta\lambda} = m \cdot N$

$$N = \frac{\lambda}{m\,\Delta\lambda} = \frac{589.6}{3 \cdot (589.6 - 589)} = 327.6$$

$\Rightarrow N > \underline{328}$

Oppgave 1 forts

d) Når lys kommer fra et optisk mindre tett medium og reflekteres på grenseflaten mot et optisk tettere medium, endres fasen med π. Kommer lyset fra et optisk tettere medium og reflekteres mot et optisk mindre tett medium, blir det ingen faseforandring.
Ved brytning blir det ingen faseforandring.

e) Reflektert stråle fra grenseflaten luft/film får faseforandring π. Når strålen går inn i filmen og reflekteres på grenseflaten film/luft, blir det ingen faseforandring.
De to reflekterte strålene får derfor destruktiv interferens når $2t = m \cdot \lambda / n$

$$2tn = m \cdot \lambda$$

der t er filmtykkelsen, n er filmens brytningsindeks og $m = 0, 1, 2, 3 \cdots$. For en mørk stripe er altså

$$t = \frac{m \cdot \lambda}{2n} = \frac{m \cdot 5{,}8 \cdot 10^{-7}}{2 \cdot 1{,}5} \, m = m \cdot 193 \text{ nm}$$

Hvis avstanden mellom stripene er a, blir kilvinkelen

$$\alpha = \frac{t_1}{a} = \frac{193 \cdot 10^{-9}}{0{,}60 \cdot 10^{-2}} = \underline{3{,}22 \cdot 10^{-5} \text{ rad}} = \underline{1{,}84 \cdot 10^{-3}\,°}$$

Filmtykkelsen ved 43. minimum:

$$y = t_{42} = 42 \cdot 193 \cdot 10^{-9} \text{ m} = 8{,}1 \cdot 10^{-6} \text{ m} = \underline{8{,}1\,\mu\text{m}}$$

Oppgave 2

a) $$M \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{M} x = 0$$

$x(t) = A\cos(\omega_0 t + \varphi)$ der $\omega_0 = \sqrt{\frac{k}{M}} = \sqrt{\frac{4.0}{0.25}} = \underline{4.0\ s^{-1}}$

Startbetingelsene:

$t=0$: $x(0) = x_0 = 0{,}1\ m$

$\frac{dx(0)}{dt} = v_0 = -0{,}4\ m/s$

$$\Rightarrow \left.\begin{aligned} x(0) &= A\cos\varphi = x_0 \\ \frac{dx(0)}{dt} &= -A\omega_0 \sin\varphi = v_0 \end{aligned}\right\} \Rightarrow \underline{\tan\varphi = -\frac{v_0}{\omega_0 x_0}}$$

$$\tan\varphi = \frac{0.4}{4.0\cdot 0.1} = 1 \qquad \Rightarrow \varphi = \underline{\frac{\pi}{4}} \text{ d: } \varphi = \underline{45^\circ}$$

$$A = \frac{x_0}{\cos\varphi} = \underline{0.1\cdot\sqrt{2}\ m} = x_0\sqrt{1+\tan^2\varphi} = \underline{\sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}}$$

$$\underline{x(t) = 0.1\cdot\sqrt{2}\cos\left(4.0\, t + \frac{\pi}{4}\right)}$$

b) $\omega = \omega_0 = \underline{4.0\ s^{-1}}$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{4.0}\ s = \underline{1.57\ s}$$

$$f = \frac{1}{T} = \frac{1}{1.57}\ s^{-1} = \underline{0.64\ Hz}$$

c) Totalenergi $E = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2 = \text{konst} = E(0)$

$$E = \frac{1}{2}\cdot 0.25\cdot 0.4^2\ J + \frac{1}{2}\cdot 4.0\cdot 0{,}1^2\ J = \underline{0.04\ J}$$

$A = x_0/\cos\varphi = \underline{0.14\ m}$

Oppgave 2 forts

d) Hastigheten for elementet dz er

$$v_z = v \cdot \frac{z}{l}$$

der v er klossens hastighet

Bevegelsesenergien for elementet dz

$$dE_m = \frac{1}{2}\rho \cdot dz \cdot v_z^2 = \frac{1}{2}\rho\, dz \cdot \frac{z^2}{l^2} \cdot v^2$$

der ρ er fjærmasse pr lengdeenhet i z-retning

$$E_m = \frac{1}{2}\frac{\rho v^2}{l^2} \int_0^l z^2\, dz = \frac{1}{6} v^2 \cdot \rho \cdot l = \frac{1}{6} v^2 \cdot m = \frac{1}{2} \cdot \frac{1}{3} m v^2$$

Den potensielle energien er uavhengig av fjærens masse

$$\Rightarrow \underline{E_{tot} = \frac{1}{2}\left(M + \frac{1}{3}m\right) v^2 + \frac{1}{2} k x^2}$$

e) Totalenergien er konstant

$$\Rightarrow \frac{dE_{tot}}{dt} = 0 = \frac{1}{2}\left(M + \frac{1}{3}m\right) \cdot 2v \cdot \frac{dv}{dt} + \frac{1}{2} k \cdot 2x \cdot \frac{dx}{dt}$$

$$v = \frac{dx}{dt} \quad \text{og} \quad \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$\frac{1}{2}\left(M + \frac{1}{3}m\right) \cdot 2 \cdot v \cdot \frac{d^2x}{dt^2} + \frac{1}{2} k \cdot 2 \cdot x \cdot v = 0$$

$$\left(M + \frac{1}{3}m\right)\frac{d^2x}{dt^2} + kx = 0 \qquad \text{Svingeligning}$$

$$\Rightarrow \omega = \sqrt{\frac{k}{M + \frac{1}{3}m}}$$

$$\underline{T = \frac{2\pi}{\omega}} = \underline{2\pi \cdot \sqrt{\frac{M + m/3}{k}}}$$

(5)

Oppgave 3.

a) Ladet partikkel i magnetisk felt.

$$\frac{mv^2}{r} = qvB_m \Rightarrow \frac{q}{m} = \frac{v}{rB_m} \quad \text{eller} \quad v = \frac{qrB_m}{m}$$

Kinetisk energi $E_k = \frac{1}{2}mv^2 = \frac{1}{2}\frac{q^2r^2B_m^2}{m} = q \cdot V_a$

$$\Rightarrow \underline{\frac{q}{m} = \frac{2V_a}{r^2 B_m^2}}$$

b) $\frac{q}{m_1} = \frac{2V_a}{r_1^2 B_m^2}$ $\qquad$ $\frac{q}{m_2} = \frac{2V_a}{r_2^2 B_m^2}$

$$\frac{m_1}{m_2} = \frac{r_1^2}{r_2^2} \Rightarrow r_2 = r_1\sqrt{\frac{m_2}{m_1}}$$

$$d = 2(r_2 - r_1) = 2r_1\left(\sqrt{\frac{m_2}{m_1}} - 1\right)$$

(Indeks 1 for ^{35}Cl og indeks 2 for ^{37}Cl)

$$d = 2 \cdot 14.6\left(\sqrt{\frac{37}{35}} - 1\right)\text{cm} = \underline{0.82\,\text{cm}}$$

$$B_m^2 = \frac{2m_1V_a}{qr_1^2}$$

$$B_m = \sqrt{\frac{2m_1V_a}{qr_1^2}} = \sqrt{\frac{2 \cdot 35 \cdot 1.66 \cdot 10^{-27} \cdot 1.0 \cdot 10^3}{1.602 \cdot 10^{-19} \cdot 0.146^2}}\,T = \underline{0.184\,T}$$

$$W = qV_a = 1.602 \cdot 10^{-19} \cdot 1.0 \cdot 10^3\,J = \underline{1.6 \cdot 10^{-16}\,J} = \underline{10^3\,eV}$$

c) $\frac{q}{m} = \frac{2V_a}{r^2 B_m^2}$ Vi ser direkte at dersom r går ned til halve verdien, må ladningen være $4e$, dvs de ekstra partiklene er det firevendige ion $^{35}Cl^{4-}$.

Oppgave 3 forts

d)

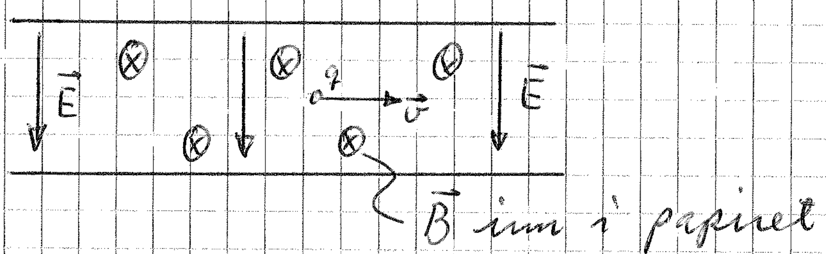
Nettovirkningen av $\vec{E}$- og $\vec{B}$-feltet må bli null

$$\vec{F} = q\vec{E} + q \cdot \vec{v} \times \vec{B}$$

$$\vec{B} \perp \vec{v} \qquad \vec{E} \perp \vec{v} \qquad \vec{B} \perp \vec{E}$$

$$qE = qvB$$

$$\underline{v = \frac{E}{B}}$$



e) Hastighetsfilteret gir hastigheten

$$v = \frac{E}{B} = \frac{3.68 \cdot 10^4}{0.5}\ \mathrm{m/s} = \underline{7{,}36 \cdot 10^4\ \mathrm{m/s}}$$

$^{35}Cl^-$ og $^{37}Cl^-$-ionene har altså samme fart

$$r = \frac{mv}{qB_m} \quad \text{idet} \quad m \cdot \frac{v^2}{r} = qvB_m$$

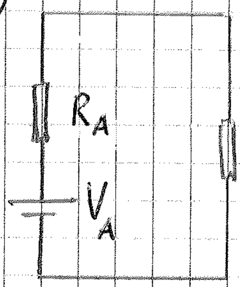
$$r_{35} = \frac{1{,}66 \cdot 10^{-27} \cdot 7{,}36 \cdot 10^4}{1.602 \cdot 10^{-19} \cdot 0{,}2} \cdot 35\ \mathrm{m} = 13{,}34\ \mathrm{cm}$$

$$r_{37} = r_{35} \cdot \frac{37}{35} = 14{,}10\ \mathrm{cm}$$

$$d = 2(r_{37} - r_{35}) = 2(14{,}10 - 13{,}34)\ \mathrm{cm} = \underline{1{,}52\ \mathrm{cm}}$$

Oppgave 4.

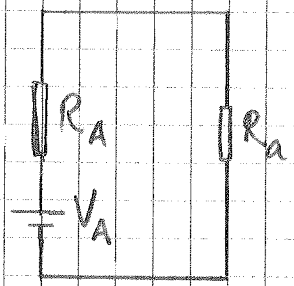
a)



$$V_P = V_A - I \cdot R_A$$

$$R_V \approx \infty \Rightarrow I \approx 0$$

$$V_A = V_P = \underline{13{,}5\,V}$$



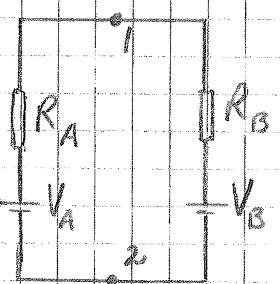
$$V_P = I R_a$$

$$V_A - IR_A = IR_a$$

$$R_a \approx 0 \Rightarrow V_A = IR_A$$

$$R_A = \frac{V_A}{I} = \frac{13{,}5}{15}\,\Omega = \underline{0{,}900\,\Omega}$$

b)

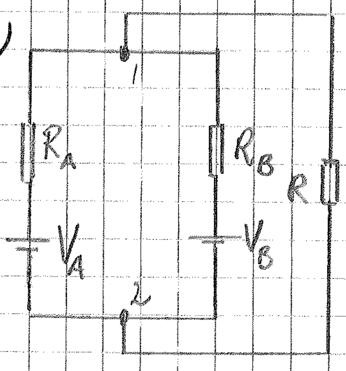


$$V_A - I_0 R_A - I_0 R_B - V_B = 0$$

$$I_0 = \frac{V_A - V_B}{R_A + R_B} = \frac{13{,}5 - 12{,}0}{0{,}900 + 0{,}800}\,A = \underline{0{,}882\,A}$$

$$V_0 = V_A - I_0 R_A = (13{,}5 - 0{,}882 \cdot 0{,}900)\,V = \underline{12{,}7\,V}$$

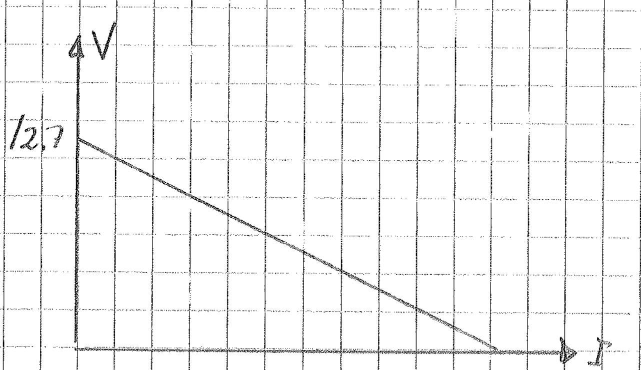
c)



$$\frac{1}{R_i} = \frac{1}{R_A} + \frac{1}{R_B} \Rightarrow R_i = \frac{R_A \cdot R_B}{R_A + R_B}$$

$$R_i = \frac{0{,}900 \cdot 0{,}800}{0{,}900 + 0{,}800}\,\Omega = \underline{0{,}424\,\Omega}$$

$$V = V_0 - R_i \cdot I = \underline{12{,}7\,V - 0{,}424 \cdot I}$$



Oppgave 4 forts

d)

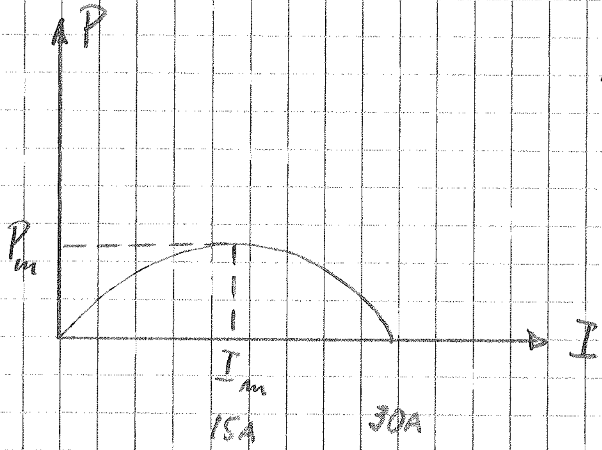
Effekt $P = R \cdot I^2 = VI = (V_0 - R_i I) \cdot I = V_0 I - R_i I^2$

$$\frac{dP}{dI} = V_0 - 2R_i \cdot I$$

$\frac{dP}{dI} = 0$ når $I = I_m$

dvs for $V_0 - 2R_i I = 0 \Rightarrow I = I_m = \frac{V_0}{2R_i} \Rightarrow \underline{R_m = R_i}$

$$P_m = R_m \cdot I_m^2 = R_i \cdot \frac{V_0^2}{4R_i^2} = \frac{V_0^2}{4R_i} = \frac{12.7^2}{4 \cdot 0.424} W = \underline{\underline{95.1 W}}$$



$$R_m = \frac{V}{I_m} = \frac{V_0 - R_i I_m}{I_m} = \frac{V_0 - R_i \cdot \frac{V_0}{2R_i}}{\frac{V_0}{2R_i}}$$

$$= \frac{\left(V_0 - \frac{V_0}{2}\right) 2R_i}{V_0} = 2R_i - R_i = \underline{R_i}$$