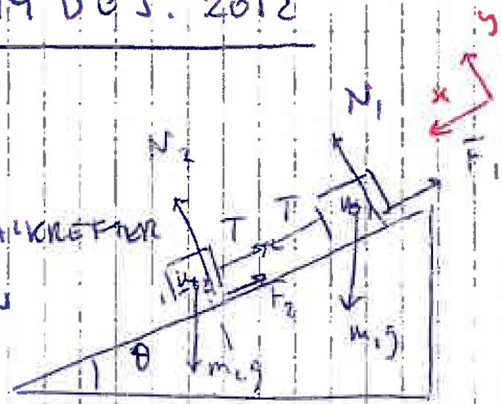
TFY4104 – 14 DES. 2012

OPPGAVE 1

1)



N_1, N_2 NORMALKREFTER

F_1, F_2 FRIKSJON

2x FRITT LEGEMEDIAGRAM

$m_1 = 0.4$ kg

$m_2 = 0.5$ kg

$\mu_1 = 0.2$

$\mu_2 = 0.1$

a) NÅR BEVEGELSE STARTER $\bar{a} \simeq 0$

$N_1 = m_1 g \cos\theta_c$; $N_2 = m_2 g \cos\theta_c$; $F_1 = \mu_1 N_1$; $F_2 = \mu_2 N_2$

$\hat{x}1$: $m_1 g \sin\theta_c + T - F_1 = 0 \Rightarrow m_1 g \sin\theta_c + T - \mu_1 m_1 g \cos\theta_c = 0$

$\hat{x}2$: $m_2 g \sin\theta_c - T - F_2 = 0 \Rightarrow m_2 g \sin\theta_c - T - \mu_2 m_2 g \cos\theta_c = 0$

$$\Sigma \quad (m_1+m_2)\cancel{g}\sin\theta_c - (\mu_1 m_1 + \mu_2 m_2)\cancel{g}\cos\theta_c = 0$$

$$\Rightarrow \tan\theta_c = \frac{\mu_1 m_1 + \mu_2 m_2}{(m_1+m_2)} = \frac{0.4\cdot 0.2 + 0.5\cdot 0.1}{0.4+0.5} = 0.135 \Rightarrow \underline{\theta_c = 9.9^\circ}$$

b) I BEVEGELSE $\mu_1 = 0.18$; $\mu_2 = 0.09$; $\theta = 20^\circ$

LEGG TIL $m_1 \cdot a$ OG $m_2 \cdot a$ TIL H.L. I $\hat{x}1$ OG $\hat{x}2$

$$\Rightarrow (m_1+m_2)\, g \sin\theta - (\mu_1 m_1 + \mu_2 m_2)\, g\cos\theta = (m_1+m_2)\, a$$

$$\Rightarrow a = g\cdot\left[\sin\theta - \frac{(\mu_1 m_1 + \mu_2 m_2)}{(m_1+m_2)}\cdot\cos\theta\right]$$

$$= 9.81\cdot\left[\underbrace{\sin 20^\circ}_{0.3420} - \underbrace{\frac{(0.18\cdot 0.4 + 0.09\cdot 0.5)}{(0.4+0.5)}}_{0.1575}\cdot \underbrace{\cos 20^\circ}_{0.9396}\right] = 1.905$$

$$\Rightarrow \underline{a = 1.9 \ m/s^2}$$

$$\underbrace{T}_{\hat{x}1 = m_1 a} = -m_1 g\sin 20^\circ + \underbrace{\mu_{1k}}_{0.18}\cdot m_1 g\cos 20^\circ + \underbrace{m_1}_{0.4}\cdot \underbrace{a}_{1.9\ m/s^2} = 0.186\ N$$

$\underline{T = 0.19\ N}$

OPPGAVE 1, FORTS.

c) $a = 1.903 \ m/s^2$ $v = v_0 + at$; $\Delta x = v_0 t + \frac{1}{2} a t^2$ FÅS LETT VED INTEGRERING AV $v = \frac{dx}{dt}$ OG $a = \frac{dv}{dt}$

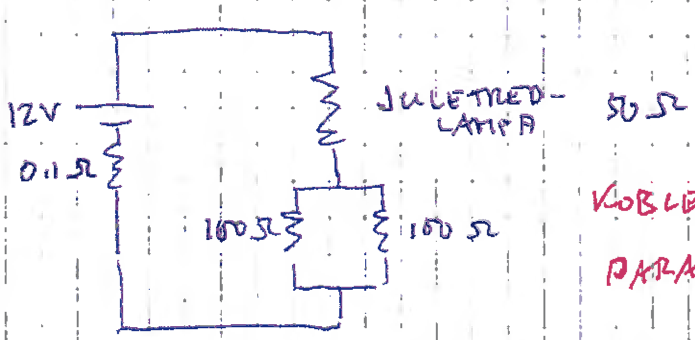
$\Delta x = 0.25 = \frac{1}{2} a t^2 \ (v_0 = 0) \Rightarrow t = \sqrt{\frac{2 \cdot 0.25}{a}} = 0.5126 \ s$

$v = v_0 + at \Rightarrow v = 1.903 \cdot 0.5126 = 0.9754 \ m/s$

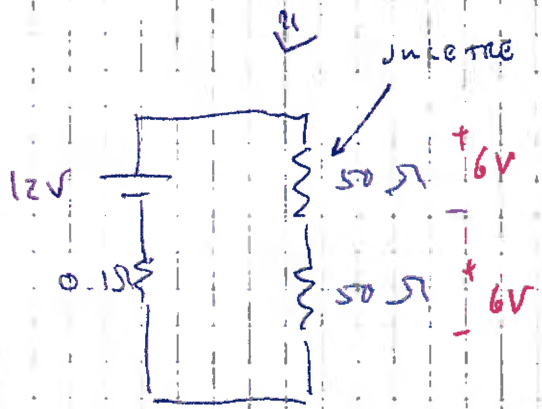
$\underline{\underline{v = 0.98 \ m/s}}$

OPPGAVE 2.

a)



KOBLE 2 100 Ω RES I PARALELL $\frac{1}{R_{tot}} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50}$



VI NEGLISJERER 0.1 Ω OG SER APPROX 6V OVER JULETREET.

$R_{tot} = 50 + 50 + 0.1 = 100.1 \ \Omega$

b) $12V = R_{tot} \cdot I \Rightarrow I = \frac{12}{100.1} = 0.120 \ A$

JA DEN KLARER DET...

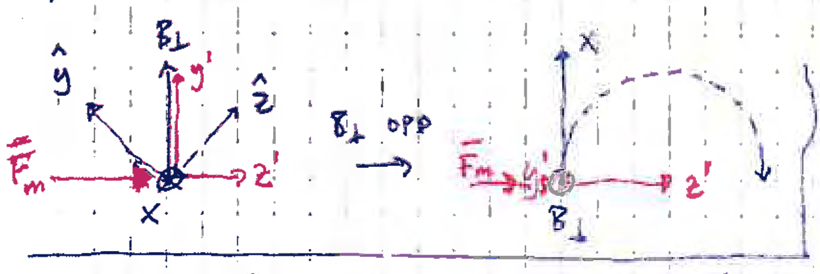
3) $v_x = 8.0\cdot10^5\ \frac{m}{s}$; $\bar B = (1,1,1)\ 0.1\ T$; $q = 2e = 2\cdot1.609\cdot10^{-19}$ C

a) $\bar F_m = q(\bar v\times\bar B) = \underbrace{\begin{vmatrix} \hat x & \hat y & \hat z \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}}\ \cdot\ \underbrace{8\cdot10^5\cdot0.1\cdot2\cdot1.607\cdot10^{-19}}_{2.57\cdot10^{-14}\ N}$

$(-\hat y+\hat z) = \sqrt2\cdot\underbrace{\left(-\frac{1}{\sqrt2}\hat y+\frac{1}{\sqrt2}\hat z\right)}_{\text{unit vector}} \longrightarrow F_m = \left(-\frac{1}{\sqrt2}\hat y+\frac{1}{\sqrt2}\hat z\right)\cdot 3.64\cdot10^{-14}\ N$

ans.. $3.6\cdot10^{-14}$ N rettet $\left(-\frac{1}{\sqrt2}\hat y+\frac{1}{\sqrt2}\hat z\right)$

b) $\bar B$ HAR KOMPONENT $B_\perp$ $\bar v$ (v_x) i yz-PLANET



MAGNETFELT $B_\perp$ VIL DREIE ION I SIRKULAER BANE, MEN, SOM DE FINNES $\bar B$ KOMPONENT LANGS $\hat x$ VIL SIRKULÆR-BANEN BLI HELIX, HVOR ER HELIX RETTET? (OPP ELLER NED?)

$B_\perp$ HAR RETNING $\frac{2}{\sqrt6}$ LANGS $\hat y'$ OG $\frac{1}{\sqrt3}$ LANGS $\hat x$ (0 LANGS $\hat z'$)

$\bar F_m$ GIR LITEN HASTIGHET LANGS $\hat z'$ ⇒ MINKNING LANGS $\hat x$

⇒ ETTER LITEN TID $\bar v = (v_x - \Delta, 0, +\Delta')$; HVA BLIR KRAFT FRA B_x?

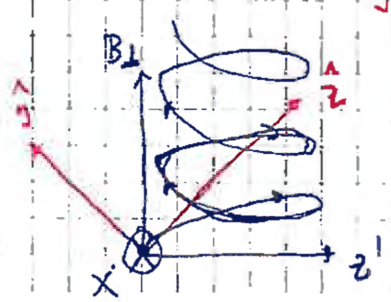
MINKER LANGS $\hat x$ ØKER LANGS z'

$$\bar F'_{m,ny} = \begin{vmatrix} \hat x & \hat y' & \hat z' \\ v_x-\Delta & 0 & \Delta' \\ \frac{1}{\sqrt3} & \frac{2}{\sqrt6} & 0 \end{vmatrix} |B| q = \left[\hat x\left(-\Delta'\cdot\frac{2}{\sqrt6}\right) + \hat y'\left(\Delta'\frac{1}{\sqrt3}\right) + \hat z'\,\frac{(v_x-\Delta)\cdot2}{\sqrt6}\right]|B|\cdot q$$

LØFTER JONER OPP-OVER $\left[\hat y'\ (B_\perp)\right]$

DREIER SIRKULAER BANE

⇒ JONENS BANE



VENSTRE SKRU

[SE KAP 26.2 I TB]

3 FORTS.

c) MED ELEKTRISK FELT KAN $\bar{F}_m$ KANSELLERES

$\bar{F}_e = q \cdot \bar{E}$; RETTET MOT $\bar{F}_m$

$\bar{F}_m$ HAR RETNING $\left(-\frac{1}{r_2}\hat{y} + \frac{1}{r_2}\hat{z} \right)$

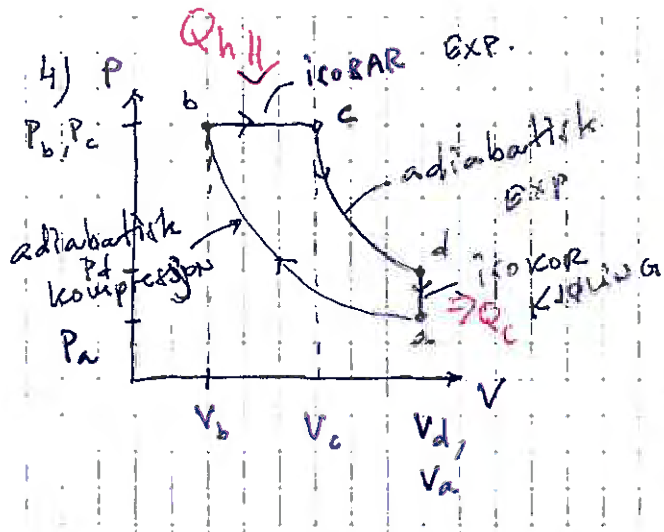
$\Rightarrow \bar{F}_e$ MÅ HA RETNING $\left(\frac{1}{r_2}\hat{y} - \frac{1}{r_2}\hat{z} \right)$

d) FELTETS STYRKE GIS AV

$$|\bar{F}_m| = q \cdot |\bar{E}| \Rightarrow |\bar{E}| = \frac{|\bar{F}_m|}{2e} = \frac{3.62 \cdot 10^{-14}\,N}{2 \cdot 1.602 \cdot 10^{-19}\,C}$$

$\Rightarrow$ FELTETS STYRKE $|E| \simeq 110 \ \frac{kV}{m}$

4)



a) SE BILDE

b) Effektivitet er gitt av

N.b. DEFINERT VARME AVGITT

$$\epsilon = 1 - \frac{Q_c}{Q_{in}}$$

IKKE NOEN VARMEOVERFØRING FOR DI ADIABATISKE PROSESSENE $c \to d$ OG $a \to b$

$b \to c \qquad dQ = C_p \, dT \Rightarrow Q_h = C_p (T_c - T_b)$

MÅ HA − TEGN DÅ Q_c' ER VARME GITT PÅ GAS

$d \to a \qquad dQ' = C_v \, dT \Rightarrow -Q_c = C_v (T_a - T_d)$

Q_{in} TILL GAS

$$\Rightarrow \epsilon = 1 - \frac{-C_v (T_a - T_d)}{C_p (T_c - T_b)} = 1 + \frac{1}{\gamma} \frac{(T_a - T_d)}{(T_c - T_b)} \quad : \text{ans.}$$

SETT INN

c) $$\frac{P_b V_b}{T_b} = \frac{P_c V_c}{T_c} \overset{P_b = P_c}{\Longrightarrow} \frac{T_c}{T_b} = \frac{V_c}{V_b} \Rightarrow T_b = T_c \cdot \frac{V_b}{V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{1}{T_c} \frac{(T_a - T_d)}{\left(1 - \frac{V_b}{V_c}\right)} \; ; \quad \frac{P_c V_c}{T_c} = \frac{P_a V_a}{T_a} \; ; \quad \frac{P_c V_c}{T_c} = \frac{P_d V_d}{T_d}$$

$$\Downarrow \qquad \qquad \Downarrow$$

SETT INN

$$T_a = T_c \cdot \frac{P_a V_a}{P_c V_c} \qquad T_d = T_c \frac{P_d V_d}{P_c V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{1}{\cancel{T_c}} \frac{\left[\cancel{T_c}\left(\frac{P_a V_a}{P_c V_c}\right) - \cancel{T_c}\left(\frac{P_d V_d}{P_c V_c}\right)\right] \times V_c}{\left(1 - \frac{V_b}{V_c}\right) \times V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{\left(\frac{P_a V_a}{P_c} - \frac{P_d V_d}{P_c}\right)}{(V_c - V_b)} \qquad P_c = P_b \Rightarrow$$

$$\epsilon = 1 + \frac{1}{\gamma} \frac{\frac{P_a V_a}{P_b} - \frac{P_d V_d}{P_c}}{(V_c - V_b)}$$

FORTS.

4) c) FORTS,

NÅ BRUK ADIABATISKE PROSESSENE $a \to b$; $c \to d$

$$P_a V_a^\gamma = P_b V_b^\gamma \Rightarrow \frac{P_a}{P_b} = \frac{V_b^\gamma}{V_a^\gamma}$$

$$P_c V_c^\gamma = P_d V_d^\gamma \Rightarrow \frac{P_d}{P_c} = \frac{V_c^\gamma}{V_d^\gamma}$$

SETT INN I ϵ $\Rightarrow \epsilon = 1 + \frac{1}{\gamma}\frac{\left(V_b^\gamma V_a^{1-\gamma} - V_c^\gamma V_d^{1-\gamma}\right)}{(V_c - V_b)}$

VI HADDE $V_a = V_d$ SETT IN

$$\epsilon = 1 + \frac{1}{\gamma}\frac{V_a^{1-\gamma}\left(V_b^\gamma - V_c^\gamma\right)}{(V_c - V_b)} \overset{\text{tegn}}{=} 1 - \frac{1}{\gamma}\frac{V_a^{1-\gamma} V_b^\gamma \left(V_c^\gamma - V_b^\gamma\right)}{V_b\left(V_c - V_b\right)} \frac{\frac{1}{V_b^\gamma}}{\frac{1}{V_b}} =$$

$$\Rightarrow 1 - \frac{1}{\gamma}\frac{V_a^{1-\gamma}}{V_b^{1-\gamma}}\frac{\left[\left(\frac{V_c}{V_b}\right)^\gamma - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} = 1 - \frac{1}{\left(\frac{V_a^{\gamma-1}}{V_b^{\gamma-1}}\right)\gamma}\frac{\left[\left(\frac{V_c}{V_b}\right)^\gamma - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} =$$

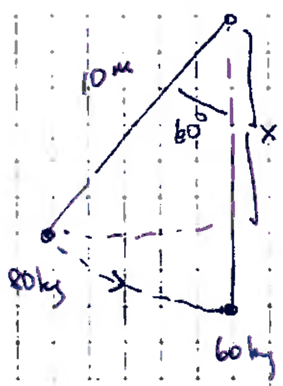
$$= 1 - \frac{1}{\gamma\left(\frac{V_a}{V_b}\right)^{\gamma-1}}\frac{\left[\left(\frac{V_c}{V_b}\right)^\gamma - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} = 1 - \frac{1}{\gamma r^{\gamma-1}}\left[\frac{\alpha^\gamma - 1}{\alpha - 1}\right] =$$

MED $r = \frac{V_a}{V_b}$ OG

$\alpha = \frac{V_c}{V_b}$

QED

5)



5 m

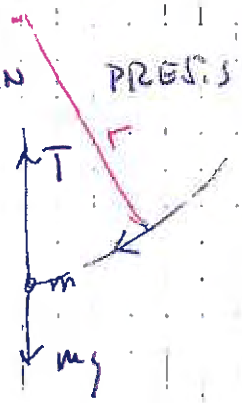
$\} = 5\ m = h$

$\frac{x}{10} = \cos 60° \Rightarrow x = 10 \cos 60° = 5\ m$

a) BRUK ENERGIPRINSIPPET: $\cancel{m}gh = \frac{\cancel{m}v^2}{2}$ $\quad v = \sqrt{2gh}$

$g = 9.81\ m/s^2$, $h = 5\ m$ $\Rightarrow$ $v = 9.9\ m/s$

b) TARZAN PRESIS FOR TREFF



$T - mg = \frac{mv^2}{r}$ $\quad (+)$

SIRKELBEVEGELSE $\Rightarrow$ CENTRIPETAL AKSELERASJON

POSITIV INN MOT SIRKELSENTRUM

$\Rightarrow T = mg + \frac{mv^2}{r} = m\left(g + \frac{2gh}{r}\right) = mg\left(1 + \underbrace{\frac{2 \cdot h}{r}}_{=1}\right) = 2mg$

SIFFRER $\quad T = 2 \cdot 80\ kg \cdot 9.81\ m/s^2 = 1570\ N \simeq 1600\ N$

c) $\underset{\text{FØR}}{m_1 v_1 + m_2 v_2} = \underset{\text{ETTER}}{(m_1 + m_2) v_3}$

$v_1 = 9.9\ m/s$ (TARZAN), $\quad v_2 = 0$ (JANE)

$v_3 = \frac{m_1}{(m_1 + m_2)} \cdot v_1 = \frac{80\ kg}{140\ kg} \cdot 9.9\ \frac{m}{s} \simeq 5.7\ \frac{m}{s}$

d) SOM (b) NÅ $m = m_1 + m_2$

$T - (m_1 + m_2) g = \frac{(m_1 + m_2) v_3^2}{r}$ $\Rightarrow T = 140\ kg \left(9.81\ \frac{m}{s^2} + \frac{5.7^2 \left(\frac{m}{s}\right)^2}{10\ m}\right)$

$\simeq 1800\ N$

e) $\frac{\cancel{(m_1+m_2)} v_3^2}{2} = \cancel{(m_1+m_2)}\, g h \Rightarrow h = \frac{v_3^2}{2g} = \frac{5.7^2\ \frac{m^2}{s^2}}{2 \cdot 9.81\ \frac{m}{s^2}} = 1.7\ m$

$\cos \alpha = \left(\frac{10 - 1.7}{10}\right) \Rightarrow \alpha = 33°$