nyp

1a)

$$\frac{f''}{f} = -k^2 \Longrightarrow f'' + k^2 f = 0$$

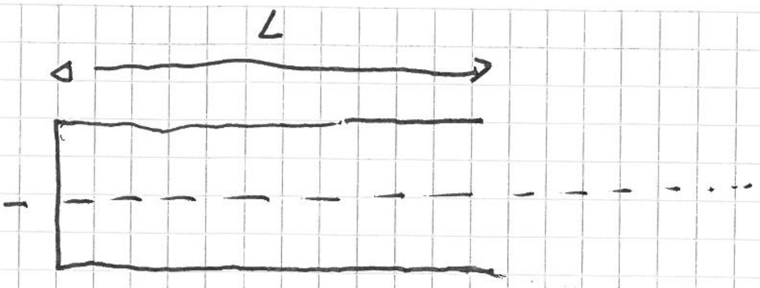
standard 2. ordens diff ligning. (masse-fjær) m/ løsn.

$$\underline{f(x) = f_0 \cos(kx + \varphi_f)}$$

$$\frac{\ddot{g}}{g} = -v^2 k^2 = -\omega^2 \Longrightarrow \ddot{g} + \omega^2 g = 0$$

m/ løsning: $\underline{g(t) = g_0 \cos(\omega t + \varphi_g)}$

1b)



$\xi(0,t) = 0$

$p(L,t) = 0$

La $\xi(x,t) = \xi_0 \cos(kx + \varphi)\cos(\omega t + \varphi')$

$\xi(0,t) = 0 \Rightarrow k\cdot 0 + \varphi = \pm\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$= (n - \frac{1}{2})\pi$

~~Velger $\varphi = 0$~~ Velger $\varphi = -\frac{\pi}{2}$ og $\varphi' = 0$

$\Rightarrow \underline{\xi(x,t) = \xi_0 \sin(kx)\cos(\omega t)}$

$\underline{p(x,t) = -B\frac{\partial \xi}{\partial x} = -B\xi_0 k \cos kx \cos\omega t}$

$p(L,t) = 0 \Leftrightarrow kx = (n - \frac{1}{2})\pi$

$\Rightarrow \xi(L,t) = \xi_0 \cos(\omega t)$

$= \underline{MAX}$

$p(0,t) = -B\xi_0 k \cos\omega t = \underline{MAX}$

- Grunntonen $\quad kL = \frac{\pi}{2} \quad (n=1)$

$$\frac{2\pi}{\lambda} L = \frac{\pi}{2}$$

$$\lambda = 4L$$

ξ profil

p-profil

(7p)

c) Fra (5) har vi at $k_n L = (n - \frac{1}{2})\pi$

videre er $\lambda_n f_n = v$ $\quad (\frac{\omega_n}{k_n} = v)$

$$\Rightarrow \quad \frac{2\pi}{\lambda_n} = \frac{(n-\frac{1}{2})\pi}{L}$$

$$\frac{f_n}{\cancel{v}} = \frac{(n-\frac{1}{2})\cdot v}{2L}$$

Løser for n:

$$\frac{2L f_n}{v} + \frac{1}{2} = n$$

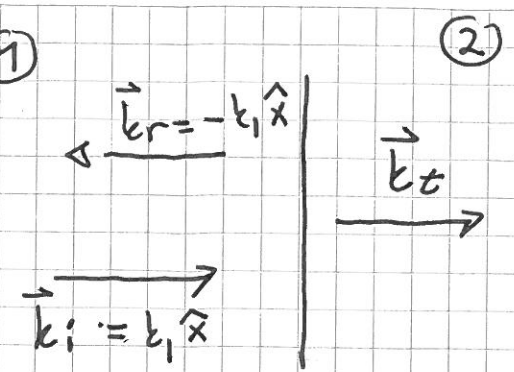
setter inn $L = 1$ m, $f_n = 577.5$ Hz og

$v = 330$ m/s

$$\Rightarrow \quad n = \frac{1}{2} + \frac{2 \cdot 577.5}{330} = 4$$

Dette er 3. harmoniske $(n = 4)$

d)



3p

i) $\xi_i(x,t) = \xi_{0i}\sin(\vec{k}_i\cdot\vec{x} - \omega t) = \xi_{0i}\sin(k_1 x - \omega t)$

$\xi_r(x,t) = \xi_{0r}\sin(\vec{k}_r\cdot\vec{x} - \omega t) = \xi_{0r}\sin(-k_1 x - \omega t)$

$\xi_t(x,t) = \xi_{0t}\sin(\vec{k}_t\cdot\vec{x} - \omega t) = \xi_{0t}\sin(k_2 x - \omega t)$

$\vec{x} = x\hat{x}$

ii) $p_i(x,t) = -B_i\dfrac{\partial\xi_i}{\partial x} = -B_1\xi_{0i}k_1\cos(k_1 x - \omega t)$

$p_r(x,t) = -B_r\dfrac{\partial\xi_r}{\partial x} = +B_1 k_1\xi_{0r}\cos(-k_1 x - \omega t)$

$p_t(x,t) = -B_t\dfrac{\partial\xi_t}{\partial x} = -B_2 k_2\xi_{0t}\cos(k_2 x - \omega t)$

$v_{pi}(x,t) = \dfrac{\partial\xi_i}{\partial t} = -\xi_{0i}\,\omega\cos(k_1 x - \omega t)$

$v_{pr}(x,t) = -\xi_{0r}\,\omega\cos(-k_1 x - \omega t)$

$v_{pt}(x,t) = -\xi_{0t}\,\omega\cos(k_2 x - \omega t)$

3p

iii) Grensebetingelser :

Kont. trykk: $p_i(0,t) + p_r(0,t) = p_t(0,t)$

$$\Rightarrow -B_1 \zeta_{0i} k_1 + B_1 k_1 \zeta_{0r} = -B_2 k_2 \zeta_{0t} \qquad (1) \quad \Big|(\cdot \cos\omega t)$$

Kont. partikkelhastighet :

$$v_{pi}(0,t) + v_{pr}(0,t) = v_{pt}(0,t)$$

$$\Rightarrow \zeta_{0i} + \zeta_{0r} = \zeta_{0t} \qquad (2) \quad \Big| \cdot \frac{\cos\omega t}{\omega}$$

(1)/$(\zeta_{0i} B_1 k_1)$ gir

$$-1 + r = -\frac{B_2 k_2}{B_1 k_1} t \qquad (3)$$

(2)/ζ_{0i} gir

$$1 + r = t \qquad (4)$$

$-(3) + (4)$:

$$2 = t\left(1 + \frac{B_2 k_2}{B_1 k_1}\right) = t\left(\frac{B_1 k_1 + B_2 k_2}{B_1 k_1}\right)$$

$$\Rightarrow \underline{t = \frac{2 B_1 k_1}{(B_1 k_1 + B_2 k_2)}}$$

setter t inn i (4)

$$\Rightarrow \quad \cancel{\text{m}}\ r = -1 + \frac{2B_1k_1}{B_1k_1 + B_2k_2}$$

$$= \frac{-B_1k_1 - B_2k_2 + 2B_1k_1}{B_1k_1 + B_2k_2} = \underline{\frac{B_1k_1 - B_2k_2}{B_1k_1 + B_2k_2}}$$

~~setter~~ Bruker at

$$v = \sqrt{\frac{B}{g}} = \frac{\omega}{k}$$

$$\Rightarrow \quad k = \omega\sqrt{\frac{g}{B}}$$

$$\Rightarrow \quad t = \underline{\underline{\frac{2\sqrt{B_1 g_1}}{\sqrt{B_1 g_1} + \sqrt{B_2 g_2}}}}$$

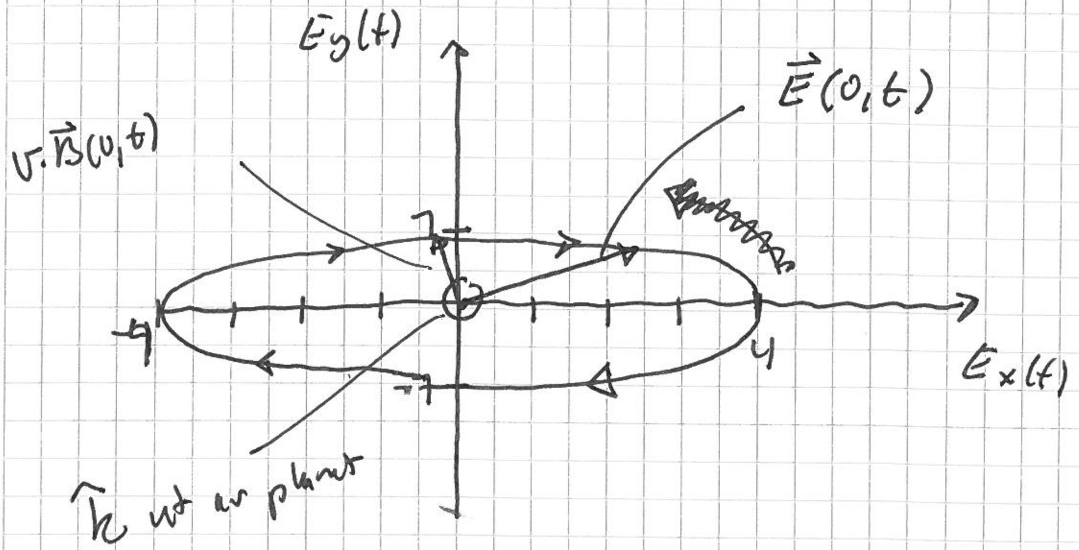
$$r = \underline{\underline{\frac{\sqrt{B_1 g_1} - \sqrt{B_2 g_2}}{\sqrt{B_1 g_1} + \sqrt{B_2 g_2}}}}$$

2 a)

$$E(0,t) = \underbrace{2\cos\omega t}^{E_x(t)}\,\hat{x} + \underbrace{\overbrace{\cos\left(\omega t + \frac{\pi}{2}\right)}^{E_y(t)}}_{-\sin(\omega t)}\,\hat{y}$$

$$\frac{E_x^2(t)}{4} + E_y^2(t) = \cos^2\omega t + \sin^2\omega t = 1$$

dette er ligning for en ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



dreie retning : Med klokka siden $E_y(t) = -\sin(\omega t)$

$\vec{B}(0,t) = \frac{1}{v}\hat{k} \times \vec{E}$ $\perp$ $\vec{E}$ feltet. Vil

følge samme ellipse, alltid $\perp$ på $\vec{E}$ feltet.

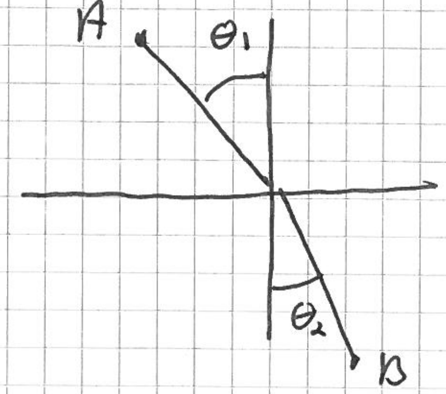
2b)

i) ~~Lyse~~ Fermats prinsipp :

3

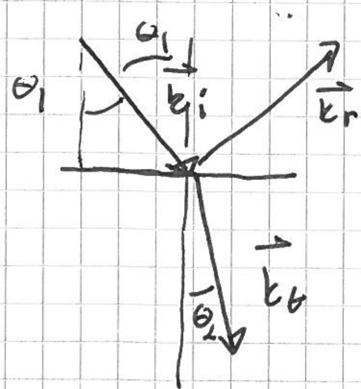
– lyset tar veien som tar kortest tid

eksempel



t_{AB} = MIN

ii)



alle k-vektor i ett plan.

4

$$k_i \sin\theta_1 = k_t \sin\theta_2$$

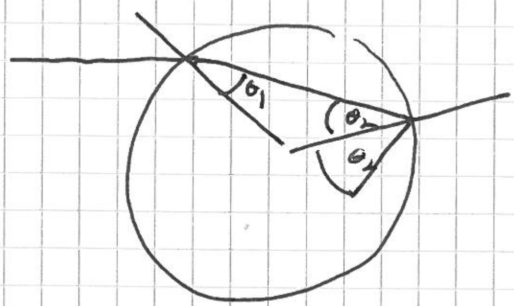
$$k_i = \frac{2\pi}{\lambda} n_1 \quad , \quad k_t = \frac{2\pi}{\lambda} n_2$$

$$\Rightarrow \quad \underline{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

c)

sollyset består av E-felt komponent $\parallel$ og $\perp$ til planet gitt i skisse (som er innfallsplanet til alle flatene)

Ved bakre flate, så er $\theta_2 \approx \theta_{Brewster}$



$\Rightarrow$ $\underline{R_p(\theta_2) \approx 0}$ (dvs. E felt-komponent $\parallel$ til flate)

Må være med!

Refleksjon på bakre flate er for $\theta_2 \approx \theta_{Brewster}$

$\Rightarrow$ lineært polarisert lys etter refleksjon.

Begrunnelse

- Feltet har komponent $\perp$ (ut av planet) til planet siden

$\underline{R_s \neq 0, \text{ mens } R_p \approx 0}$.

3a) Løsning med determinant metoden:

(i) setter inn prøveløsning $\theta = Ae^{i\omega t}$ (ev. $A\cos\omega t$)

(siden en egenmode er definert ved at begge massene svinger med samme frekvens.), i lign. 3.1.

Ligt (3.1) ~~blir~~ blir da

$$-\omega^2\theta_1 + \left(\frac{k}{m}+\frac{g}{l}\right)\theta_1 = \frac{k}{m}\theta_2$$

$$-\omega^2\theta_2 + \left(\frac{k}{m}+\frac{g}{l}\right)\theta_1 = \frac{k}{m}\theta_1$$

Setter opp på formen $\bar{\bar{A}}\,\bar{\theta} = 0$.

$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0$$

$$\alpha = -\omega^2 + \left(\frac{k}{m}+\frac{g}{l}\right), \quad \beta = -\frac{k}{m}.$$

denne har løsning for $\det(\bar{\bar{A}}) = 0$

$$\Rightarrow \quad \alpha^2 - \beta^2 = 0$$

$$\left(-\omega^2 + \left(\frac{k}{m}+g/l\right)\right)^2 - \left(\frac{k}{m}\right)^2 = 0$$

$$-\omega^2 \mp \left(\frac{k}{m}+\frac{g}{l}\right) = \pm\frac{k}{m}$$

$$\Rightarrow \quad \omega' = \omega^+ = \underline{\underline{\sqrt{\frac{g}{l}}}}$$

$$\omega'' = \omega^- = \underline{\underline{\sqrt{\frac{2k}{m} + \frac{g}{l}}}}$$

ii) $\theta_1(0) = \theta_2(0)$: symmetrisk mode

$\theta_1(0) = -\theta_2(0)$: antisymmetrisk mode

(trenger ikke si hvilken som er ω' eller ω'')

b)

Ligningene i 3.1 beskriver kun N2 for hver av massene, det vil si

$$\sum F = m\ddot{\theta}_1 = -\left(\frac{k}{m} + \frac{g}{l}\right)\theta_1 + \frac{k}{m}\theta_2$$

Legger da ganske enkelt $F_0 \cos\omega t$ til høyre side i denne ligningen:

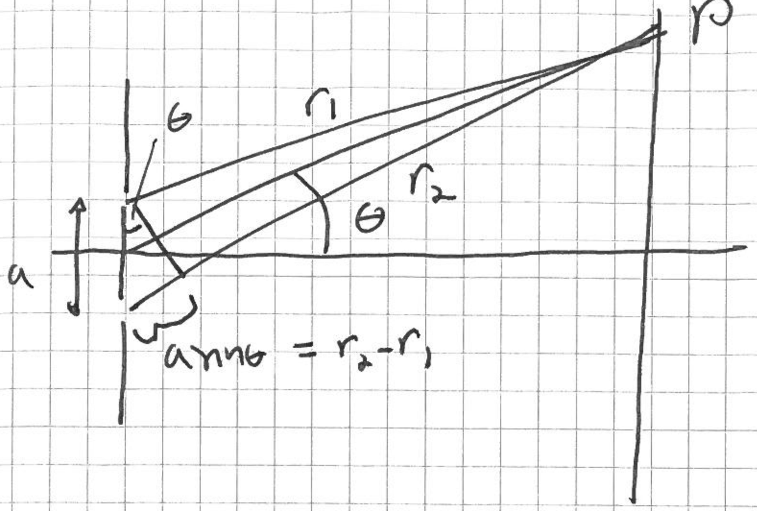
$$\Rightarrow \quad \ddot{\theta}_1 + \left(\frac{k}{m} + \frac{g}{l}\right)\theta_1 = \frac{k}{m}\theta_2 + F_0\cos\omega t$$

$$\underline{\ddot{\theta}_2 + \left(\frac{k}{m} + \frac{g}{l}\right)\theta_2 = \frac{k}{m}\theta_1}$$

Ved $\omega = \omega'$ forventer vi resonans, og utslagene blir store uten annen demping. Lignings-settet 3.1 blir videre ~~dog~~ ugyldig p.g.a. at $\sin\theta \neq \theta$.

4)

a)



Fraunhofer $\Rightarrow$ $\simeq \parallel$ stråler

$$E(P) = \left(\frac{E_0}{r_0} e^{ikr_1} + \frac{E_0}{r_0} e^{ikr_2} \right) e^{-i\omega t}, \quad r_1 \approx r_2 \approx r_0$$

$$= \frac{E_0}{r_0} e^{-i\omega t} e^{ikr_1} \Big(1 + \underbrace{e^{ik(r_2 - r_1)}}_{e^{ika\sin\theta}} \Big)$$

$$= 2 \cdot \frac{E_0}{r_0} e^{-i\omega t} e^{ikr_1} e^{ika\sin\theta/2} \underbrace{\left(\frac{e^{ika\sin\theta/2} + e^{ika\sin\theta/2}}{2} \right)}_{\cos(ka\sin\theta/2)}$$

$$\langle I(P) \rangle \sim \cancel{2} \frac{E_0^2}{r_0^2} \cos^2(ka\sin\theta/2) \cdot \varepsilon_0 c$$

der $\langle \cos(k(r_1 + a\sin\theta/2) - \omega t) \rangle = \frac{1}{2}$

Oppgitt $I_0 = \frac{1}{2} \varepsilon_0 c E_0^2$

$$\Rightarrow \langle I(P) \rangle \sim 4 \frac{E_0^2}{r_0^2} \cos^2(\alpha) \qquad \text{QED}$$

b) Omhyldningskurven $\frac{\sin x}{x}$, har første nullpunkt for $x = \pi$.

$\Rightarrow$ leser ut at $\theta \approx 0.42 \pm 0.2$

$$\frac{1}{2} k b \sin\theta = \pi \quad , \quad k = \frac{2\pi}{\lambda}$$

$$b = \frac{\lambda}{\sin\theta} \approx \frac{514 \text{ nm}}{\sin(0.42 \text{ rad})} \approx \underline{1.32\ \mu m}$$

Interferensmaksima for $\alpha = n\pi$, $n = 0, \pm 1, \pm 2, \pm 3 ..$

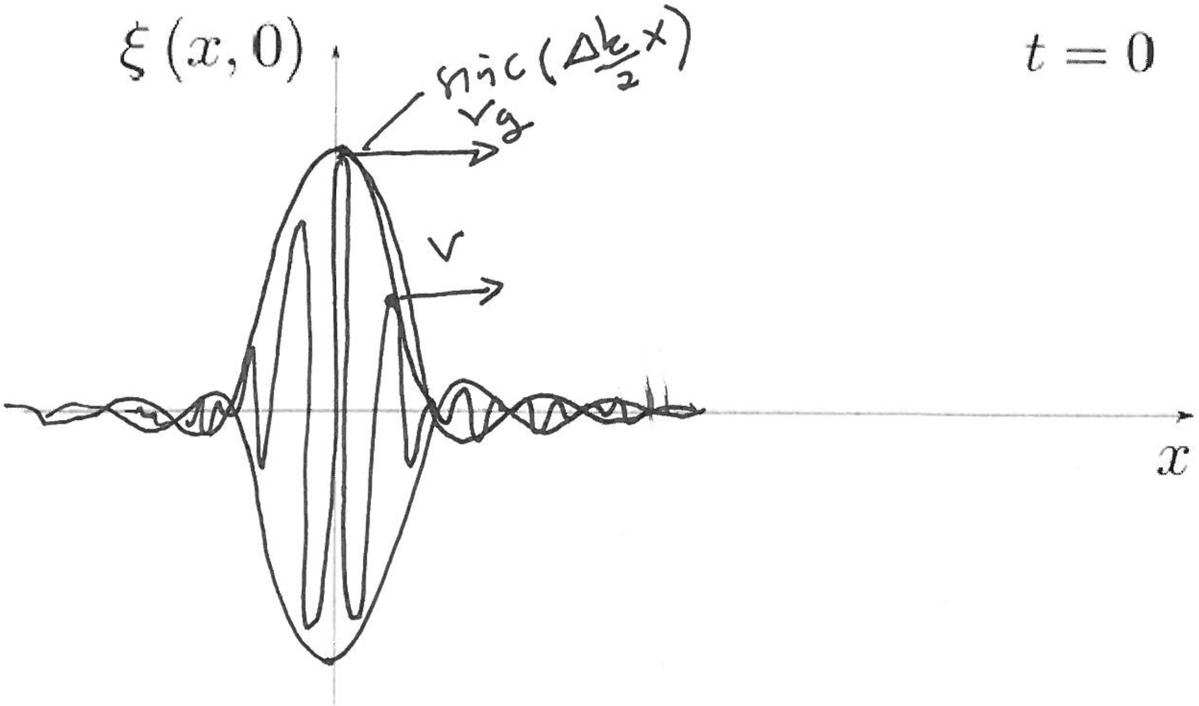
Fra sentrum til 6. maksimum: $\theta_6 \approx 0.25$

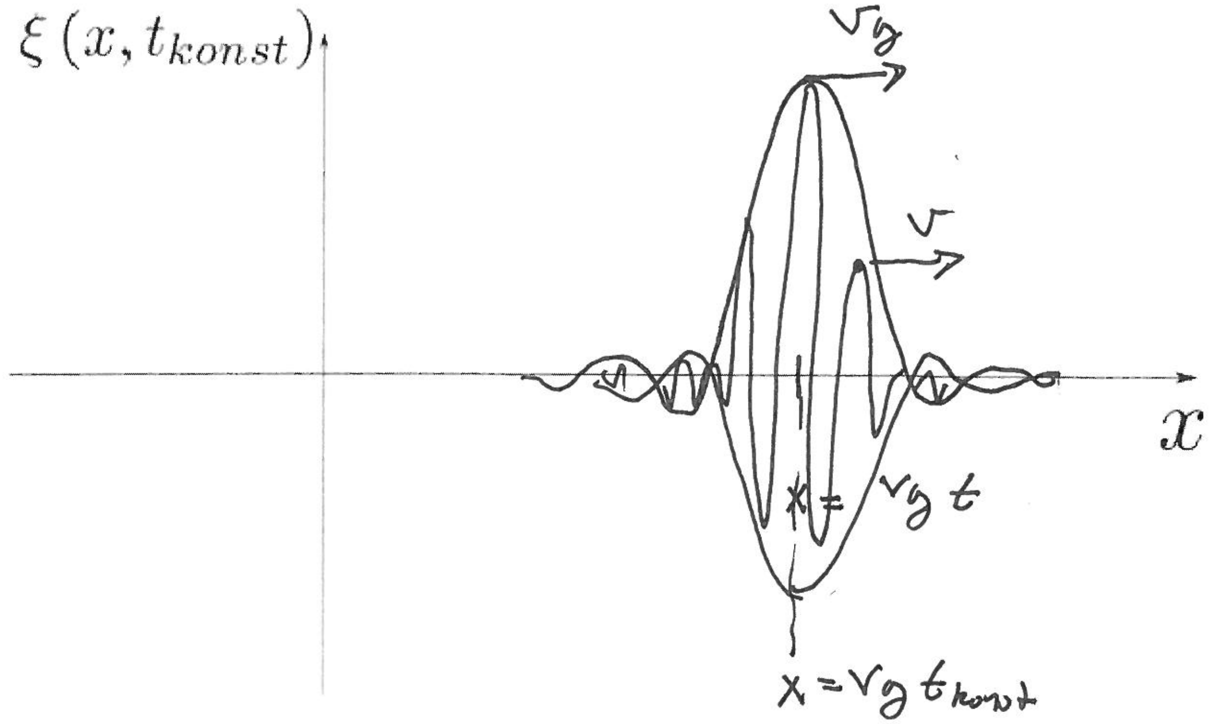
$$\frac{1}{2} k a \sin\theta_6 = 6\pi$$

$$a = \frac{6\lambda}{\sin\theta_6} \approx \frac{6 \cdot 514 \text{ nm}}{\sin(0.25)} \approx \underline{12.46\ \mu m}$$

Svar Ark, Oppgave 5a. i

Kandidatnr.:..........................
Dato:......... Side: 15
Antall Ark:..........................





$t_{konst} = konstant > 0$

5a) ii) $\frac{\sin x}{x} \rightarrow 1$ for $x \rightarrow 0$

siden oppgitt at $\xi_0 \Delta k$ = konstant

$\Rightarrow$ bølgen blir en ren harmonisk bølge med én frekvens (og ett bølgetall).

b)

$$\omega^2 = gk \tanh kd$$

$$v = \frac{\omega}{k} = \frac{\sqrt{gk \tanh kd}}{k}$$

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(gk \tanh kd\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(gk \tanh kd\right)^{-\frac{1}{2}} \cdot \left\{ g \tanh kd + gk \underbrace{\frac{d}{dk}\left(\tanh kd\right)}_{\operatorname{sech}^2 kd \,\cdot\, d} \right\}$$

$$v_g = \frac{1}{2}\left(gk \tanh kd\right)^{\frac{1}{2}} \left\{ g \tanh kd + gkd \operatorname{sech}^2 kd \right\}$$

Ikke dispersiv $\Longleftrightarrow$ $v = v_g =$ konst. uavhengig av k

$$\tanh kd \to kd \Rightarrow v = \sqrt{gd}$$

$$\tanh kd = kd - \frac{(kd)^3}{3!} + \dots$$

$$\Rightarrow (kd)^2/\cancel{3!} \ll \cancel{kd}\; 3! \Rightarrow d^2 \ll \frac{\lambda^2}{4\pi^2}$$

6a) $P = 1 \text{ Watt} = 1 \text{ J/s}$

(i) $E = hf = \dfrac{hc}{\lambda}$

$\dfrac{\Delta N}{\Delta t}$ = antall fotoner per sekund

$$= \frac{P}{E} = \frac{\lambda}{hc} \cdot P$$

$$= \frac{1400 \times 10^{-9}\,[\cancel{m}]}{6.626 \times 10^{-34}\,[\cancel{J} \cdot s] \cdot 3 \times 10^{8}\,\cancel{m}/\cancel{s}} \cdot 1\,\cancel{J}/s$$

$$= 7 \times 10^{18}\,[s^{-1}]$$

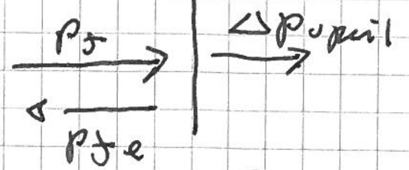
(ii) $F = \dfrac{\Delta p}{\Delta t}$

Fotonets impuls:

$E^2 = p_f^2 c^2 \iff E = p_f c$

$p_f = E/c$

Impulsbevaring:



$\Delta p = 2 p_f = \dfrac{2 \cdot E}{c}$ per foton

$$F = \frac{\Delta N}{\Delta t} \cdot \Delta p = \frac{P}{E} \cdot \frac{2 \cdot E}{c} = \frac{2P}{c} = \frac{2W}{3 \times 10^{8}\,[m/s]} = 6.67\,nN$$

6b) I' er He-atomets referanse system.
I' - laboratorie systemet har en hastighet $-v$

$$f' = f\left(\frac{c-(-v)}{c+(-v)}\right)^{\frac{1}{2}}$$ (formel fra appendix)

$$= f\left(\frac{(c+|v|)}{(c-|v|)}\right)^{\frac{1}{2}}, \quad |v| = 0.7c$$

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\left(\frac{c+|v|}{c-|v|}\right)^{\frac{1}{2}}$$

$$\lambda' = \lambda\left(\frac{(c-|v|)}{(c+|v|)}\right)^{\frac{1}{2}} = 1400 \times 10^{-9}\,m \cdot \left(\frac{0.3}{1.7}\right)^{\frac{1}{2}}$$

$$= 588 \text{ nm}$$

(merknad He-I absorbsjonslinje er ved 587 nm)