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INSTITUTT FOR FYSIKK

Contact during the exam:
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EXAM: TFY4245 FASTSTOFF-FYSIKK VK

Monday 19. May 2008
Tid: kl 09.00-13.00

Allowed exam material: Alternative C
Standard pocket calculator
Rottman: Mathematical Formula (all language editions)
Barnett og Cronin: Mathematical Formula

The exam consists of:

1. The first page (the present page) which must be delivered with answers to the multiple choice questions.
2. 3 "normal" Problems 1, 2 and 3 (Appendix A)
3. One set of multiple choice questions, Problem 4 (Appendix B)

The three "normal" problems count altogether 50%, and the multiple choice questions count altogether 50%. Only ONE of the alternatives A-D must be marked for each of the 20 multiple choice questions. Correct answer gives one point, wrong answer gives zero points.

Answers to the multiple choice questions in Appendix B:

Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer												

Question	13	14	15	16	17	18	19	20
Answer								

Problem 1. Linear response:

1a)

Consider a physical system that is forced out of equilibrium by applying a time (t) dependent external force, $\sigma(t)$, to it.

For linear response, the response field perturbation of the system is:

$$\gamma(t) = \alpha\, \sigma(t) \qquad (1)$$

where α is a linear response function which is characteristic of the specific system under consideration.

Assuming that the system returns to equilibrium, $\gamma(t = \infty)$, at a rate, $d\gamma(t)/dt$, which increases proportionally with the magnitude of the perturbation, $\gamma(t)$, we may write:

$$d\gamma(t)/dt = (\gamma(t = \infty) - \gamma(t))/\tau \qquad (2)$$

where the system characteristic time, τ, is the constant of proportionality between the rate and the perturbation.

τ contains information about the dynamics of physical processes in the system that couple to the external field σ.

Equation (1) represents a simple relaxation process (Debye relaxation).

Assume, that we apply a time dependent periodic force to the system, such that:

$$\sigma(t) = \sigma_0 \exp(-i\omega t) \qquad (3)$$

where ω is the applied frequency, and σ_0 is the force amplitude.

The resulting linear response can be written:

$$\gamma(t\,) = \alpha(\omega)\, \sigma(t) = \gamma_0 \exp(-i\omega t + i\delta(\omega)) \qquad (4)$$

where $\delta(\omega)$ is the phase difference between the force and the response, and and γ_0 is the response amplitude.

- Use the above equations in order to derive an expression for the complex frequency dependent linear response function

  $$\alpha(\omega) = \exp(i\delta(\omega)) = \gamma(t)/\sigma(t) = \alpha'(\omega)+i\alpha''(\omega) \qquad (5)$$

  for the case of a simple relaxation process like the one in Equation (2). In addition sketch both $\alpha'(\omega) = \mathrm{Re}(\alpha(\omega))$ and $\alpha''(\omega) = \mathrm{Im}(\alpha(\omega))$ for this case. ($i = \sqrt{-1}$)

1b)

During a cyclic experiment like the one described in a), the power per unit volume provided by the external force may be written:

$$dw/dt = \mathrm{Re}\alpha(t)d(\mathrm{Re}\gamma(t))/dt \qquad (6)$$

where Re means Real part.

- Show that Equation (6) may be rewritten in the form

  $$dw/dt = -(1/2)\, \sigma_0^2\omega\alpha'(\omega)\sin(2\omega t) + \sigma_0^2\omega\alpha''(\omega)\cos^2(\omega t) \qquad (7)$$

  and discuss the meaning of the two terms in Equation (7).
- Use Equation (7) to write down an expression for the time-averaged power take-up by the system, and discuss the meaning of the result.

Note typo in Eq. (6), should say $dw/dt = \mathrm{Re}\sigma(t)d(\mathrm{Re}\gamma(t))/dt$

1c)

- For general linear response functions $\alpha(\omega)$, (i.e. not necessarily simple relaxation), derive the following Kramers-Kronig relations between the real and imaginary parts respectively:

$$\alpha'(\omega_0) = (1/\pi)P\int[\alpha''(\omega)/(\omega-\omega_0)]d\omega \quad (8)$$

$$\alpha''(\omega_0) = -(1/\pi)P\int[\alpha'(\omega)/(\omega-\omega_0)]d\omega \quad (9)$$

where the integral limits are from $-\infty$ to $+\infty$, and the principal value, P is defined as

$$P\int[\alpha(\omega)/(\omega-\omega_0)]d\omega = \lim_{\delta\to 0} \{\int_1[\alpha(\omega)/(\omega-\omega_0)]d\omega + \int_2[\alpha(\omega)/(\omega-\omega_0)]d\omega\} \quad (10)$$

where the limits of integral 1 ($\int_1$) are between $-\infty$ and $\omega_0-\delta$, and those of integral 2 ($\int_2$) are between $\omega_0-\delta$ and ∞.

- Discuss the physical contents of the Kramers-Kronig relations.

Problem 2. Nematics and the Isotropic-Nematic transition:

2a)

- What is the general meaning of, and the general definition of, an order parameter in the context of phase transitions?
- Discuss what is meant by a nematic phase.
- Show that the order parameter for nematic ordering most suitably is chosen to be

$$S_2 = \langle(3\cos^2\theta - 1)/2\rangle \qquad (10)$$

- and discuss the meaning of the θ -angle, as well as the meaning of the averaging denoted by $\langle\ \rangle$ for this case.

2b)

Landau theory is a thermodynamic theory describing phase transitions, and this theory considers the Helmholtz free energy $f(\eta,T)$, where η is the order parameter, and T is the temperature: Landau theory assumes that near the phase transition temperature T_C, it is possible to expand the Helmholtz free energy in powers of η, thus

$$f(\eta,T) = f_0 + c_2\eta^2 + d_3\eta^3 + c_4\eta^4 + c_5\eta^5 + c_6\eta^6 + ...+ c_n\eta^n + ... \qquad (11)$$

Assume that $c_2(T) = b(T - T_C)$, where b is a positive constant, and also assume that $c_3, c_4, c_5, c_6, ..., c_n, ...$ are constants independent of T.

- Consider the case when $d_3 = -c_3$ is negative, c_4 is positive, and $c_n = 0$ for $n > 4$, and argue why such a non-zero c_3 is necessary in order to describe the isotropic-nematic transition, i.e. we are considering the Landau-de Gennes model for the nematic transition which then is:

$$f(S_2,T) = f_0 + c_2S_2^2 - c_3S_2^3 + c_4S_2^4 \qquad (12)$$

- Use Equation (12) to show that the isotropic to nematic transition in this model is a first order phase transition, and that at the transition temperature T_{ni}, $S_2(T_{ni}) = C_3/(2C_4)$.
- Use Equation (12) to calculate $S_2(T)$ for $T < T_{ni}$.

2c)

The Maier-Saupe theory for nematics introduces a molecular field which acts to orient the rod-like molecules of standard nematics. This molecular field has the form

$$u(\theta) = -u_0S_2((3\cos^2\theta-1)/2) \qquad (13)$$

- Discuss the meaning of the molecular field and its terms as given in Equation (13)

Boltzmann statistics gives the orientation distribution function $w(\theta,\phi)$ for this case, i.e.

$$w(\theta,\phi) = \exp(u(\theta)/k_BT)/Z \qquad (14)$$

where k_B is Boltzmann's contant, T is the temperature, and the normalization factor Z is the partition function.

- Use Equation (14) to show how it can be calculated from this model that $S_2(T_{ni}) = 0.44$
- Discuss the conceptual difference between the Landau-de Gennes theory introduced in b) and the present Maier-Saupe approach.

Problem 3. Superconductivity:

3a)

- Sketch and discuss magnetization curves (i.e. magnetization **M** vs magnetic field **H**) for type I and type II superconductors respectively.

3b)

Using Ginzburg-Landau wave mechanics, the superconducting current density (for Cooper pairs) may be written

$$\mathbf{j} = -2e|\psi|^2(h\nabla\theta(\mathbf{r})/2\pi + 2e/\mathbf{A})/2m_e \qquad (15)$$

where $\psi(r) = |\psi(\mathbf{r})|\exp(i\theta(\mathbf{r}))$ is the wavefunction with phase $\theta(\mathbf{r})$, 2e is the Cooper pair charge, $2m_e$ is the Cooper pair mass, and **A** is the vector potential, related to the magnetic field through $\nabla\mathbf{x}\mathbf{A} = \mathbf{B}$.

- Show how you can combine Equation (15) with the Maxwell equation $\nabla\mathbf{x}\mathbf{B} = \mu_0\mathbf{j}$, in order to derive the London equation

  $$\nabla^2\mathbf{B} = \mathbf{B}\lambda_L^{-2} \qquad (16)$$

  where λ_L is the London penetration depth, defined as

  $$\lambda_L^{-2} = \mu_0\rho_S(-2e)^2/(2m_e) \qquad (17)$$

  and the Cooper pair density $\rho_S = |\psi|^2$.
- Discuss and sketch the meaning of λ_L.

3c)

The Ginzburg-Landau theromodynamic theory for superconductivity assumes Equation (11) for superconductivity with $|\psi|$ as the orderparameter, and b and c_4 positive, $c_3 = 0$ and $c_n = 0$ for $n > 4$.

- Use these assumptions to calculate the equilibrium value $\rho_S = |\psi|_{eq}^2$ for $T < T_C$.

3d)

Type II superconductors may involve spatial variations of ρ_S throughout the sample, i.e. Equation (11) may not be applicable, since it involves a spatially homogenous order parameter.

In the Ginzburg-Landau-Abrikosov theory for superconductors, Equation (11) is replaced by this expression for the free energy:

$$g(|\psi|,T) = g_0 + b(T-T_C)|\psi|^2 + c_4|\psi|^4 + (h/8\pi m_e)^2|\nabla\psi|^2 + \text{(magnetic field dependent terms)} \qquad (18)$$

- Use dimensional analysis together with Equation (18) to show that there must be a length describing spatial variations of $\rho_S = |\psi|^2$ in the sample, and that this length, ξ_G, has the form

  $$\xi_G \propto (T_C - T)^{-1/2} \text{ for } T < T_C \qquad (19)$$

ξ_G is the coherence length.

- Discuss the meaning of ξ_G in relation to λ_L derived in b) above, in the context of type II superconductors.

Note typo in Eq.(15), should say $\mathbf{j} = -2e|\psi|^2(h\nabla\theta(\mathbf{r})/2\pi + 2e\mathbf{A})/2m_e$

Problem 4. Multiple choice questions:

1. The pair-distribution function $g_2(r)$ for an isotropic atomic amorphous solid or a liquid display a series of maxima and minima vc r. The minima signify:

a) Preferred positions of shells of atoms around a chosen reference atom
b) Intensity maxima of scattered x-rays from the liquid, where r is the sample to detector distance
c) The distribution of a liquid subjected to an electric field
d) None of the above

2. Bragg scattering peaks from a periodic structure may display finite widths because of:

a) Infinite sample size, i.e. infinite number of scattering planes?
b) Thermally exited phonons?
c) Only due to finite instrument resolution?
d) None of the above?

3. Which of the following type of radiation has the smallest penetration depth into materials:

a) X-rays?
b) Electrons?
c) Neutrons?
d) All radiaton (a),b),c)) have the same penetration depth?

4. The intensity I(q) of scattered x-rays from an isotropic sample is expressed as $I(q) \propto N\,S(q)$, where N is the number of particles in the scattering volume, S(q) is the interference function, and $q = (4\pi\sin\theta)/\lambda$ is the magnitude of the scattering vector. In order to derive these expressions for I(q) and q , we have assumed that:

a) The scattering is inelastic, i.e there is a frequency shift in the scattering process from the primary to the scattered beam
b) The atoms are located at Bravais lattice positions
c) The atoms do not move during the scattering processes
d) None of the above

5. A self-avoiding random walk in 3 dimensions will create an open object with a fractal dimension D of:

a) $D = 3$
b) $D = 2$
c) $D < 1$
d) None of the above

6. Smectic order is the term used for:

a) Positional order, but no orientational order
b) Isotropic order
c) Orientational order, but no positional order
d) None of the above

7. A pure elastic solid is described by:

a) The Newtonian flow law: stress = viscosity*shear-rate
b) Hooke's law: stress = modulus*strain
c) Time-delayed relaxational behavior
d) An loss modulus that has a peak at the time characteristic for plastic flow

8. The term anelastic behavior refers to:

a) Instantaneous response
b) Time-delayed response
c) Viscous response
d) None of the above

9. By orientational polarization, we mean the induced macrocopic polarization is a result of the sum of

a) Permanent dipoles in the material that may reorient in an external electric field
b) Dipoles solely induced by an external electric field
c) Piezoelectric domains
d) None of the above

10. By distortional polarization, we mean the induced macrocopic polarization is a result of the sum of

a) Permanent dipoles in the material that may reorient in an external electric field
b) Dipoles solely induced by an external electric field
c) Piezoelectric domains
d) None of the above

11. Piezoelectricity in a material means that:

a) There is a quadratic relationship between elastic and electric fields in a material?

b) There is a linear relationship between the elastic and electric fields in a material?

c) There is no coupling between elastic and electric fields in a material?

d) None of the above?

12. Diamagnetism:

a) Diamagnetism is the term used for magnetism in materials with permanent magnetic dipoles
b) The atomic diamagnetic susceptibility is independent of the number of electrons in the atom
c) Diamagnetism is present in all materials
d) None of the above?

13. Paramagnetism:

a) Paramagnetism is present in all materials
b) The paramagnetic susceptibility for a sample with localized magnetic moments is proportional to the temperature
c) The paramagnetic susceptibility is in general small (compared to the diamagnetic susceptibility for example) and and negative
d) None of the above?

14. For second order phase transition, near the critical temperature T_C, critical fluctuations may become important for the observed behavior. Which of the following statements is true?

a) Critical fluctuations are included in the classical Landau theory?
b) Critical fluctuations are caused by infinitely large restoring forces preventing emergent dynamic and spontaneous regions of finite order parameter near T_C?
c) Critical fluctuations are considered to be objects with a size proportional to the lattice spacing of the material?
d) None of the above?

15. Ferroelectricity is the results of

a) Dipole-dipole interactions
b) Distortional polarization only
c) Piezoelectric couplings in the material
d) None of the above

16. Ferromagnetism is basically a result of:

a) Classical interaction between magnetic dipoles?
b) Quantum mechanical exchange forces between spins of neighboring atoms?
c) Quantum mechanical exchange of phonons resulting in paired electrons?
d) None of the above?

17. Nuclear Magnetic Resonance (NMR) can be observed

a) Only in ferromagnetic materials
b) Only for spin 3/2 nuclei
c) In all materials independent of nuclear spin states
d) None of the above

18. Cooper pairs:

a) Are bosons?
b) Are formed by pairs of electrons bound together by exchanging photons?
c) Are localized to within the unit cell?
d) None of the above?

19. In a Dynamic Light Scattering (DLS) experiment, the dynamic structure factor S(**q**,t) is measured, where **q** is the scattering vector. Typical scattering curves from samples made from a dilute suspensions of Brownian diffusing colloidal spheres follow a simple relaxational behavior of the kind $S(q,t) = \exp(-t/\tau(|\mathbf{q}|))$, where the relaxation time $\tau(|\mathbf{q}|) = C\,|\mathbf{q}|^2$. The proportionality constant C is

a) $C = D_S$, the self diffusion constant?
b) $C = 1/D_S$, the inverse of the self diffusion constant?
c) C has no physical meaning, its just a fitting parameter to the data?
d) $C = 1/\eta$, the inverse of the viscosity of the suspending liquid?

20. The widths of the Brillouin doublet peaks measured in Rayleigh - Brillouin experiments provide information about:

a) Lifetimes of sound waves?
b) Propagation velocities sound waves?
c) Lifetimes of heat waves?
d) Propagation velocities of heat waves?