SIF40AH/DIF4997
Nano-particle and polymer physics I
SOLUTION of EXERCISE 8

Eq. (x.x) refers to version AM11sep02 of lecture notes: "Nano-particle and polymer physics". Equations pertinent to this exercise you will find in Ch. 5.1.3

A)

$$\begin{aligned}
\text{Displacement :} \quad x(t) &= x_0 \sin \omega t & (1)\\
\text{Friction force :} \quad F^{(\mathrm{h})} &= -\zeta \dot{x} = -6\pi\eta R \omega x_0 \cos \omega t & (2)\\
\text{Inertia force :} \quad F^{(\mathrm{m})} &= m\ddot{x} = -\frac{4}{3}\pi R^3 \rho_\mathrm{s} \omega^2 x_0 \sin \omega t & (3)
\end{aligned}$$

where η is the viscosity of the fluid, ρ_s is the mass density of the sphere and ζ is the friction coefficient for spheres (Stokes law). Then

$$\frac{|F^{(\mathrm{m})}|}{|F^{(\mathrm{h})}|} = \frac{\frac{4}{3}\pi R^3 \rho_\mathrm{s} \omega^2 x_0}{6\pi\eta R \omega x_0} = \underline{\frac{2}{9}\frac{\rho_\mathrm{s}}{\eta}\omega R^2}. \tag{4}$$

B) The result shows that $|F^{(\mathrm{m})}| \leq |F^{(\mathrm{h})}|$ when $\omega \leq \frac{9}{2}\frac{\eta}{\rho_\mathrm{s}}\omega R^{-2}$. For $\rho_\mathrm{s} = 2000$ kg/m^3 and $\eta = 1,0 \cdot 10^{-3}$ Ns/m^2, this yields

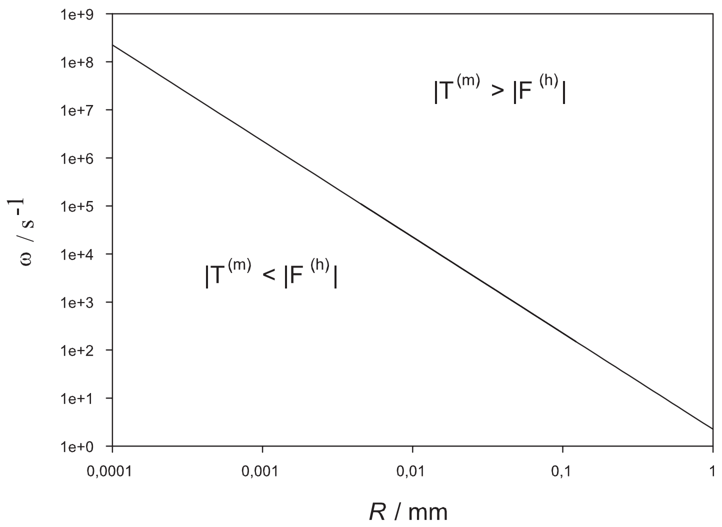
$$\omega \leq \frac{\frac{9}{4}\ \mathrm{s}^{-1}}{\frac{R}{\mathrm{mm}}}. \tag{5}$$

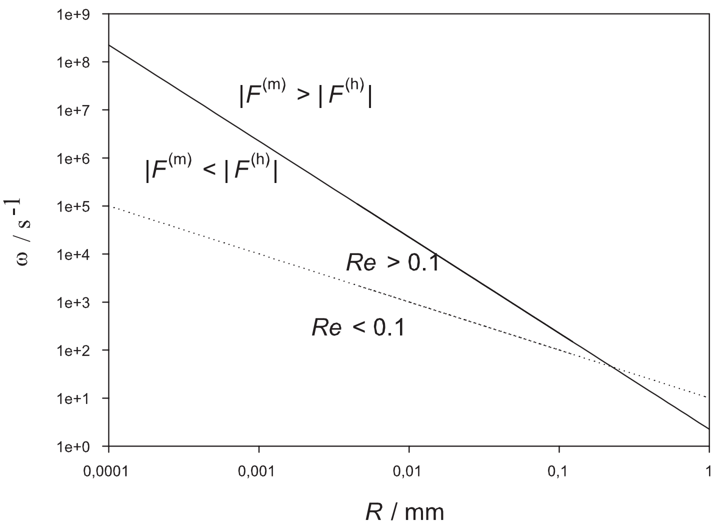
A log-log-plot of $\omega = \frac{\frac{9}{4}\ \mathrm{s}^{-1}}{\frac{R}{\mathrm{mm}}}$ as function of R is shown on next page. In the graph note that for small particles $|F^{(\mathrm{m})}| \leq |F^{(\mathrm{h})}|$ also for very rapid oscillations.

C) The Reynold number is defined (Eq. 5.7) $Re := \frac{\rho_0}{\eta}|\dot{x}|R$ and is an indicator whether the flow is laminar or not. For $Re \leq 0.1$ the Stokes law is well satisfied. For the particle described above and with $x_0 = 1\ \mu$m we get

$$\begin{aligned}
Re = R\frac{\rho_0}{\eta}\omega x_0 &\leq 0.1 \\
\omega &\leq \frac{1}{10}\frac{\eta}{x_0\rho_0}\cdot\frac{1}{R} = \frac{10\ \mathrm{s}^{-1}}{\frac{R}{\mathrm{mm}}}. & (6)
\end{aligned}$$

A log-log-plot of $\omega = \frac{10\ \mathrm{s}^{-1}}{\frac{R}{\mathrm{mm}}}$ as function of R is shown below (dotted line). Note that the condition for laminar flow is fulfilled also for very rapid oscillations when the particles are very small.


1e+9
1e+8
1e+7
1e+6
1e+5
1e+4
1e+3
1e+2
1e+1
1e+0
ω / s-1
|T(m) > |F(h)|
|T(m) < |F(h)|
0,0001
0,001
0,01
0,1
1
R / mm


1e+9
1e+8
1e+7
1e+6
1e+5
1e+4
1e+3
1e+2
1e+1
1e+0
ω / s-1
|F(m) > | F(h)|
|F(m) < | F(h)|
Re > 0.1
Re < 0.1
0,0001
0,001
0,01
0,1
1
R / mm