SIF40AH/DIF4997
Nano-particle and polymer physics I

EXERCISE 6

In this numerical exercise, choose a random-number generator from a numerical library, make your own or apply the one being a part of your computer.

A) Generate a certain number of d-dimensional vectors where the components are numbers from the random-number generator.

Imagine that you in the d-dimensional vector space have a number of sphere-shells all with centers in origo and with the same shell thickness. Make a table showing the relative number of generated vectors falling inside the individual shell of spheres, as a function of the length of the d-vector of the respective shell. Compare the result of the simulations with what is expected if the random-number generator were ideal.
Hint: See the demo-library of Press et al. "Numerical Recipes."

B) Use the approximate method for numerical transformations of random numbers to make an algorithm estimating random numbers with the distribution

$$p(y) = \exp\{-y\}$$

and compare the result of your simulations with the ideal analytical result.

The **Box-Muller algorithm** to generate random Gaussian numbers is defined:

Assume x_1 and x_2 are uniformly distributed on [0,1]. The transformations

$$y_1(x_1, x_2) = \sqrt{-2\ln x_1}\cos(2\pi x_2)$$

$$y_2(x_1, x_2) = \sqrt{-2\ln x_1}\sin(2\pi x_2)$$

yields a Gaussian distribution for both y_1 and y_2, that is

$$y_i = \frac{1}{\sqrt{2\pi}}\exp\{-y_i^2/2\}$$

C) Make a program which generates Gaussian distributed random numbers by implementing the Box-Muller algorithm. Compare your result with the ideal analytical result.