Exercise O-1. Imaging

1. Positive lens L_1.

A positive lens will focus parallel rays (i.e. a plane wave) a point behind the lens. This point is the focal point or focus and it is located in the focal plane. As the focus is a concentration of light behind the lens we can place a screen there to locate the focus - we have a **real** focus. The distance from the (thin) lens to the focal point is the **focal length f** -it is positive.
In most optical systems we may reverse the direction of propagation. This means that rays coming from a point source in the focal plane will emerge as **parallell rays** behind the lens - another way of describing this is that the spherical wave coming from the point source is transformed into a plane parallel wave by the lens (for explanatory figures - see Appendices at end of text).

The set up we are going to use do investigate the imaging properties of a positive lens is shown on figure 1.
A transparent object - *objekt* - is back-illuminated by a lamp- *belysning.* The object is placed in front of the lens to be measured -*linse.* A flat mirror, *speil,* is positioned behind the lens – the mirror should be slightly tilted. By moving the lens and the mirror along the optical axis (the bench), we find a position where we observe a sharp image of the object on the white area next to the transparency - the object is image back onto itself. This is possible only if the rays incident on the mirror are parallel. This means that the transparency is located in the front focal plane of the lens and we can determine its focal length directly by reading the positions on metric scale of the optical bench.

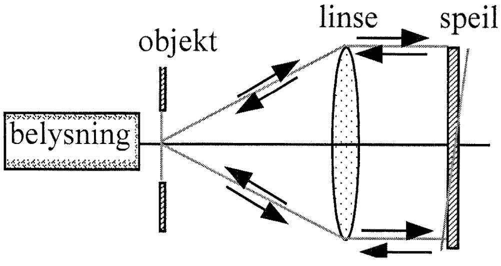


Figure 1
(*belysning* – illumination lamp, *objekt* – object, *linse* – lens, *speil* – mirror)

If you feel uncertain about the imaging process in general, for example the difference between real and virtual images, you should take time to remove the mirror and "play" with the lens.

For example put the lens close to the object and look into the lens for the virtual image.
Where is the virtual image located? (check with the lens formula if you are unsure)
Is the virtual image upright or upside down?
When the object is located in the front focal plane of the lens the angular size of the image is constant when we change our observation distance from the lens. Why?? (hint: where is the image located)

Move the lens while looking at the image – convince yourself that you also can observe the real image with your eye. **Is the real image upright or upside down?**

Where must the eye of the observer be positioned to observe the real image and why is the eye position normal to the optical axis also critical?
Use the diffuser (matte screen) to locate the real image.
Find the object and image distances for a couple of object-image combinations and put the distances into the lens formula to find the focal length.
Do the distances give the same focal length as found above?

2. Negative lens.

A negative lens will diverge parallel rays so that the rays seems to be coming from a point in front of the lens (see drawing in Appendices). This point is the focal point or focus and it is located in the focal plane - note that the focus is in front of the lens as are the incoming rays. We cannot place a screen there to obtain a focal point - we say that we have a **virtual focus**. The distance from the (thin) negative lens to the focal point is the **focal length f** which is negative.
As the negative lens does not provide a real image we have to use an extra positive lens to determine the focal length of the negative lens as will be explained with reference to figure 2.

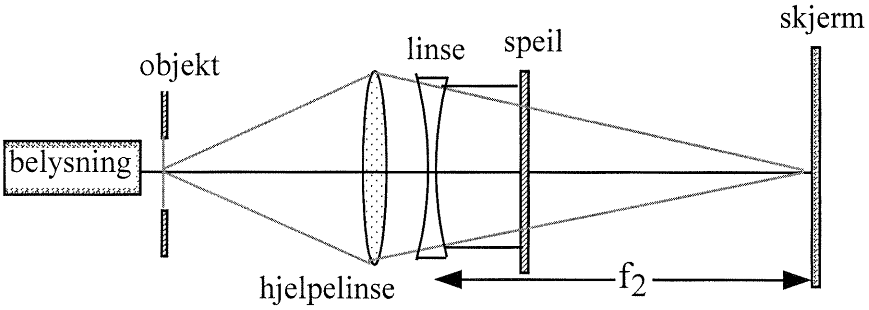


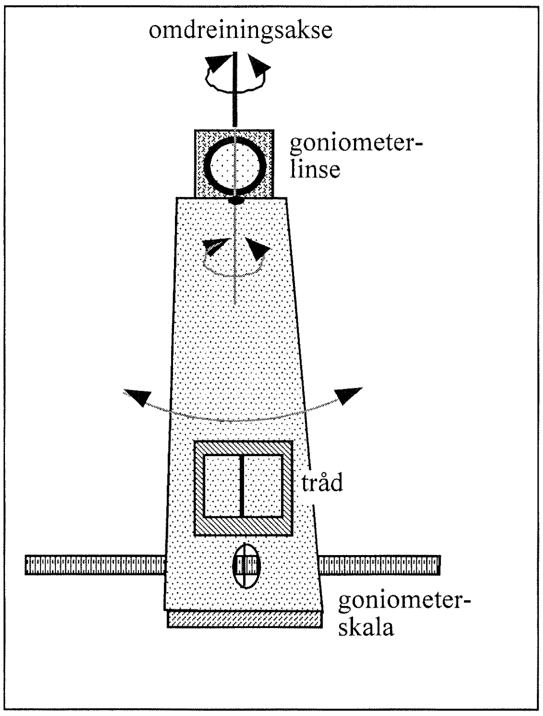
Figure 2
(*belysning* – illumination, *objekt* – object, *hjelpelinse* – help lens, *speil* – mirror, *skjerm* - screen)

In figure 2 positive lens from figure 1 is used as a "help" lens -*hjelpelinse* - and is placed in front of the negative lens -*linse* - to be measured . The mirror is placed behind the negative lens
If rays from an object point is imaged through the positive lens to the focal plane of the negative lens, the rays will emerge as parallel rays. *(think about this a while – this is not easy to understand unless you turn the argument around – parallel rays coming on to a negative lens has to emerge as diverging rays which seems to originate from the focus of the lens).*
Using the same argument as in figure 1, the rays after reflection from the mirror will create a sharp image on the transparency.

During the experiment we move the lenses and the mirror as a unit along the bench until we observe this image. When it is found we remove the mirror and the negative lens **after noting the position of the lens**. Using the screen we find the image of the object through the help lens. This image plane is according to figure 2 also the focal plane for the negative lens in the original set up.

The focal length f_2 of the negative lens is then equal to the distance between the screen and the position where the negative lens was standing

3. Angle measurement of lens parameters (the Goniometer).



The goniometer (from Latin: gonia =angle) is shown on figure 3 to the left and will be explained in the laboratory.

Omdreiningsakse – rotation axis,
goniometerlinse –goniometer lens,
tråd – wire,
skala - scale

3.1. Single lens measurement.

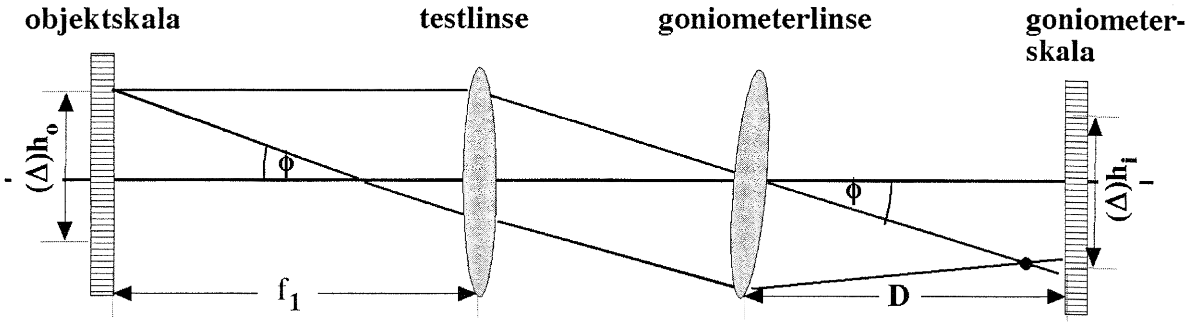


Figure 4

objektskala-object measuring scale, *linse(system)* - lens (system) to be measured, *goniometerlinse* -goniometer lens, *goniometerskala*-goniometer measuring scale, *tråd*-fine wire, *observatør*-observator

We use the goniometer as shown on figure 4. The object is a small measuring scale which enables us to determine dimensions in the object. We rotate the arm of the goniometer which makes the fine wire in the back focal plane of the goniometer lens move over the image of the object scale. It is essential that the object scale is imaged exactly in plane of the wire. We assure this by using the parallaxis effect as will be explained at the lab.

If we measure a displacement $(\Delta)h_0$ in the object plane and displacement $(\Delta)h_i$ on the goniometer scale, we have the following relation from figure 4.

$$2\,\mathrm{tg}\phi = \frac{h_o}{f_1} = \frac{h_i}{D} \qquad (1) \qquad (D = 40\ \text{cm})$$

Use the goniometer to determine the focal length of lens 1.

2.2. Focal length of a system. Principal planes.

The principal planes -*hovedplan* - H_1 and H_2 have (hopefully) been explained in class and are drawn on figure 5. Further information about the principal planes can be found in the Appendices.

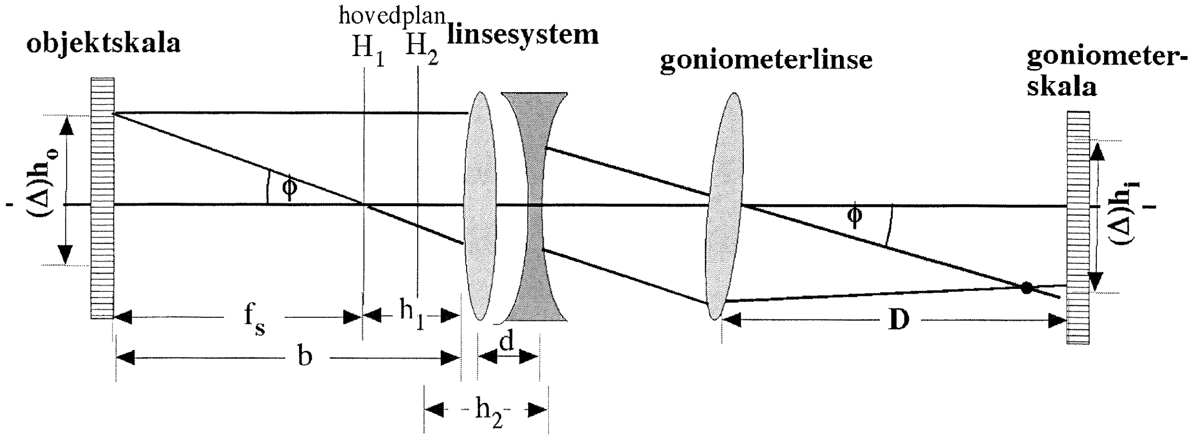


Figure 5

Depending on the form and position of the lenses in a system the principal planes can be located in different positions. We shall here look at system consisting of the positive and negative lens used earlier and we will find that the principal planes can be located outside the system. The great advantage of the principal planes is that general lens formula is still valid for a lens system if the focal lengths, object- and image-distances are referred to the distance from the principal planes.

We place the lens system in the goniometer as shown on figure 5. We use a special double optical holder for the lenses. The procedure is the same as for the single lens.

Measure the focal length f_s for the lens system for $d = 3$ cm where d is the distance between the positive and negative lens.
Measure the distance b between the object and the front lens (here L_1) and find h_1 where the principal plane H_1 is located in object space relative to the front lens.
Interchange lens 1 and 2 and find the position h_2 to the principal plane H_2.
Check the agreement between theory and measurements when:

$h_1 = (-f_s/f_2)d$ og $h_2= (-f_s/f_1)d$

The lensmaker´s formula

The focal length for a lens system can directly be calculated when we know the focal lengths and positions for the single lenses in the system. For a system consisting of two lenses with focal lengths f_1 and f_2 placed in distance d, we find the focal length of the system f_S to be:

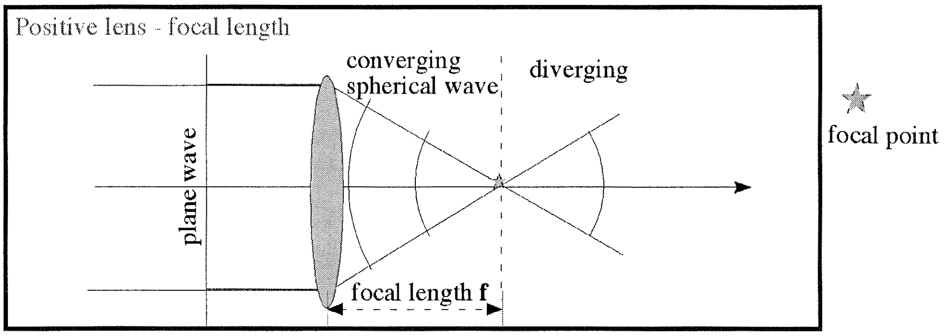
$$\frac{1}{f_s} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow f_s = \frac{f_1 f_2}{f_1 + f_2 - d} \qquad (2)$$

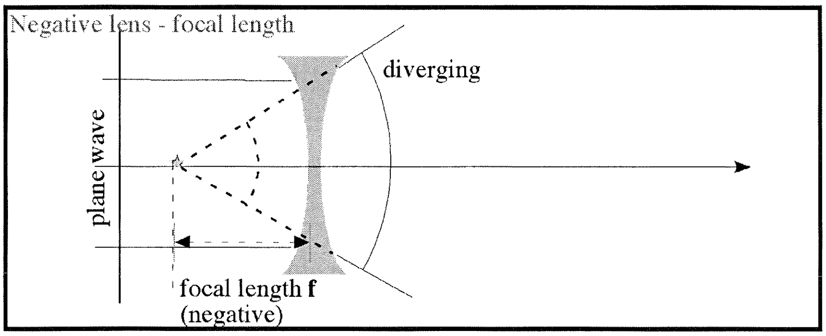
Here the individual focal lengths are given by their appropriate signs while d is positive.

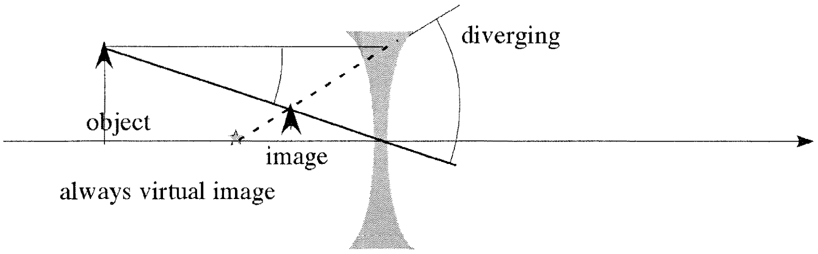
Check the focal length of the system using equation (2)

APPENDICES.

Focus definition and image formation

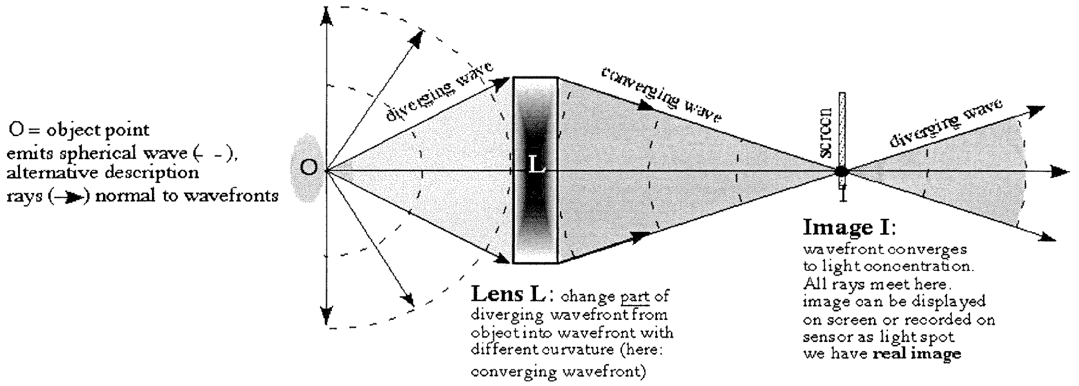




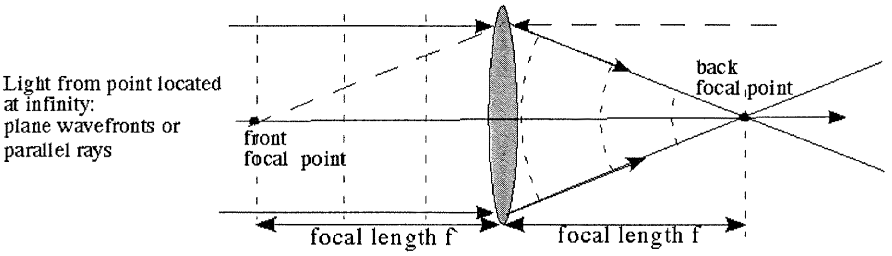


VEDLEGG A1

PHYSICAL EXPLANATION OF IMAGING

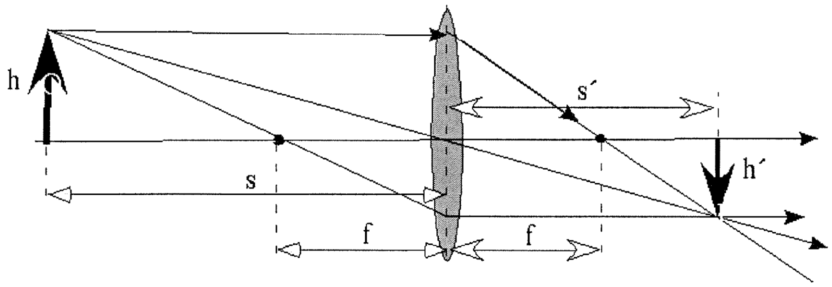


DEFINITION OF FOCAL LENGTH



Focal point is image of object located at infinite distance. Depending direction of incoming wave we get front/back focal points - the corresponding focal lengths f and f´are the same when we have the same medium on both sides of lens.

GENERAL IMAGING



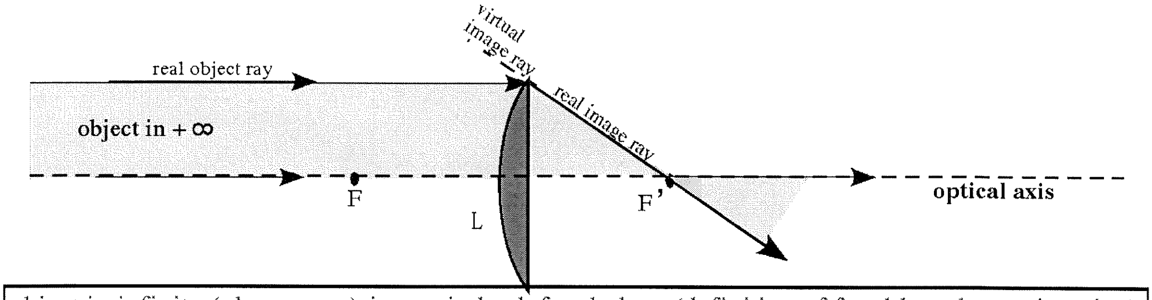
Construction of image I from object O:

Draw ray parallel to optical axis - from definition above of focal length this ray has to cross axis behind lens in **back** focal point. Draw another ray through **front** focal point - due to same definition this ray has to come out as a ray parallel to axis behind lens. Where these two rays meet is image of top of object. (You can also draw a third ray towards center of lens - if lens is **thin** this ray goes through without change of direction).
The object part which lays on the axis has to imaged on the axis and in the same plane - image is given.

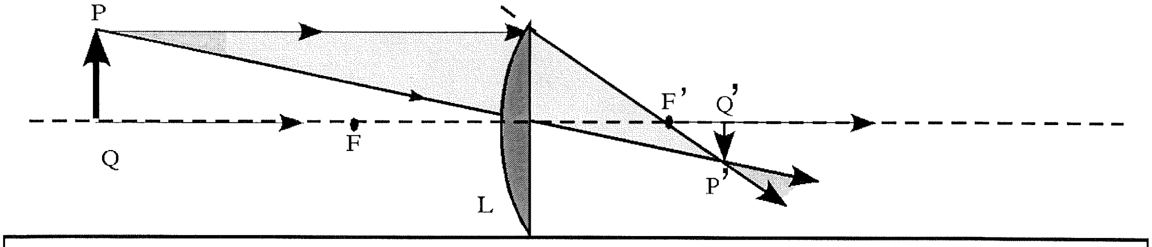
Calculation of image location and magnification:

We have several identical triangles which gives the following equations:
I) $h/s = h'/s'$ (from which we get the **magnification** $M = h'/h = s'/s$. M is here negative because the image is upside down)
II) $h/f = h'/(s' - f)$
Combining and arranging gives the **lens equation**:
$1/s + 1/s' = 1/f$
where s is positive to the left of the lens, s´ is positive to the right and f is positive to the right.

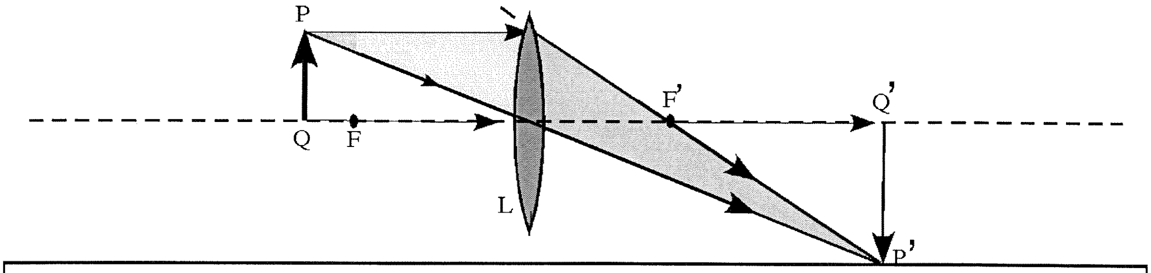
Modes of operation and applications for a positive lens(system)



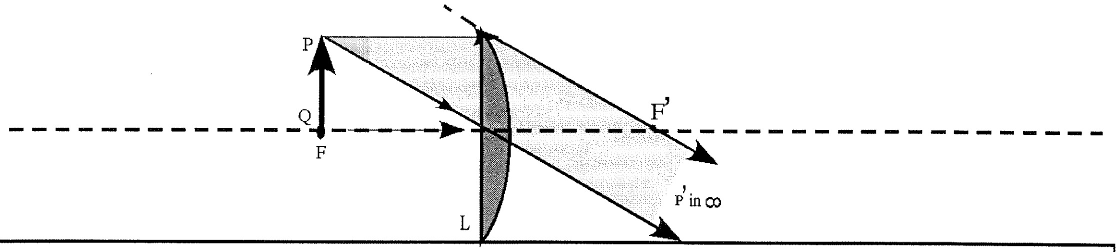
object in infinity (plane wave)-image in back focal plane.(definition of focal length, star imaging)



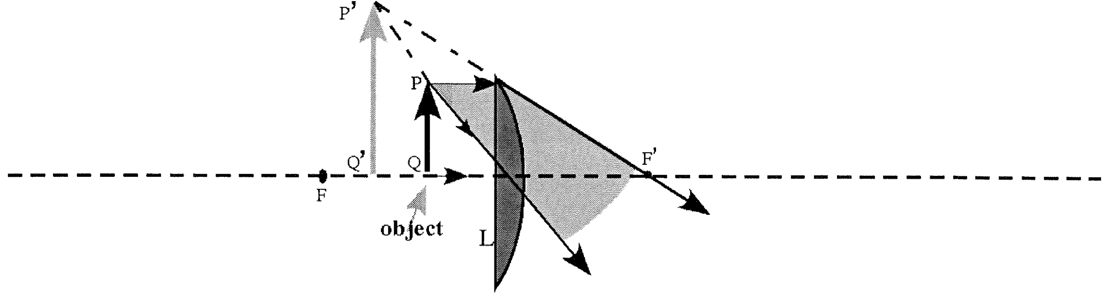
object outside 2f. Demagnified, inverted real image (telescope objective, normal camera use)



object between f and 2f. Magnified, inverted real image (microscope objective, projectors)



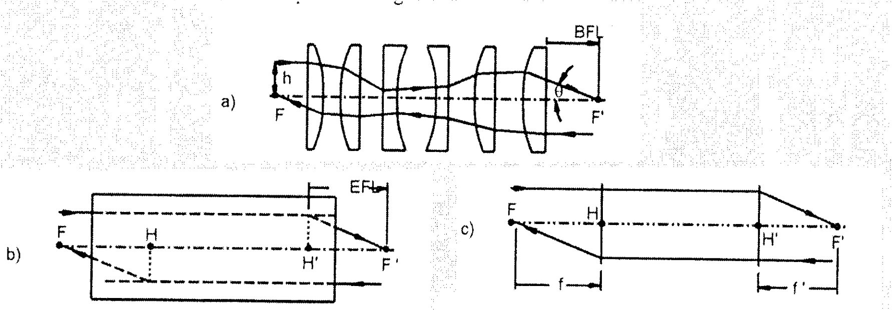
object in f. Image in infinity (final image of telescopes, microscopes)



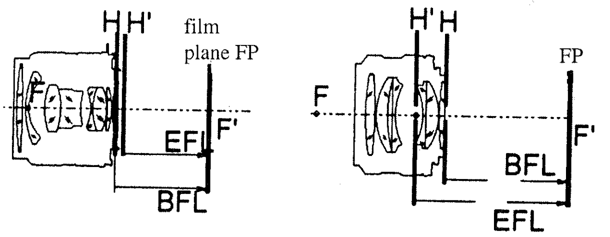
object inside f. Virtual, noninverted, magnified image (magnifying glass, oculars)

Note that the lens has been shaped and oriented to give minimum image aberrations

PRINCIPAL PLANES - definitions and positions for some commercial lenses

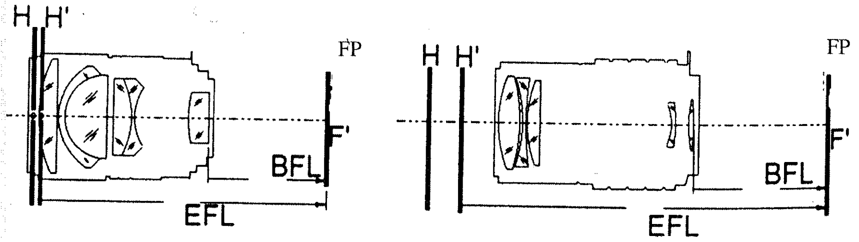


Principal planes and focal lengths for a complex lens system
a) ray path from infinity to F and F´.
b) black box representation where two planes H and H´are found by intersections of incident and emergent rays.
c) lens system replaced by the two planes



35mm-f:2 wide angle

50mm-f:1.8 normal lens



135mm-f:2 telephoto

200mm-f:4 telephoto

Principal planes and focallengths for representative camera lenses.

BFL- distance from last lens surface to film plane
EFL- effective focallength