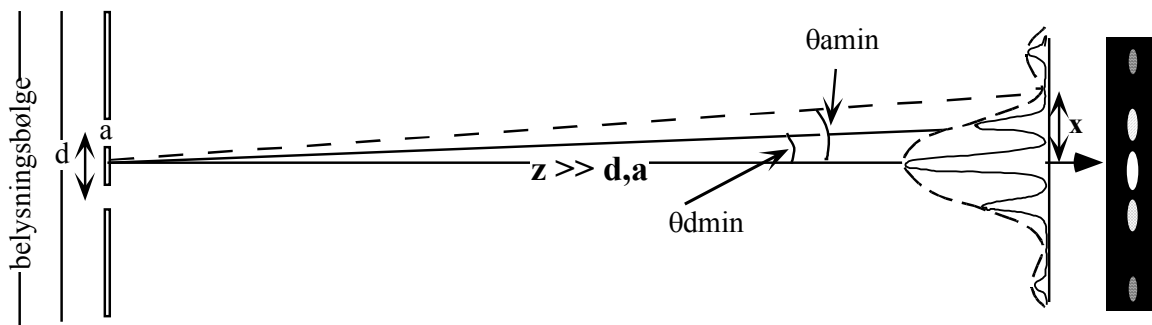


## Lab # O -4.

### Far field diffraction (gratings)

In this lab. we are going to study and record far field diffraction patterns from slits and threads. The recordings are done by a computer guided photo-detector. We are especially going to look at the transitions - diffraction -> diffraction x (multiple) interference. When several wavelengths are used in the illumination , the measurements are done in a spectrometer.



For små vinkler, dvs. når  $z \gg a$  og  $d$ :  $\sin \theta \approx \theta \approx \tan \theta = x/z$

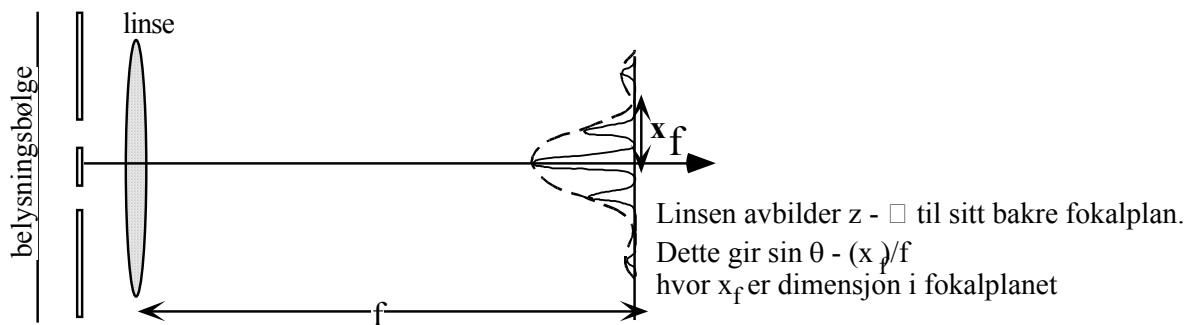


Figure 1 shows the relationship between dimensions and angles in diffraction experiments. The upper drawing shows the far field as observed by free propagation (at the extreme right is the distribution of intensity as you will observe in the lab,). The lower drawing shows the use of a lens to bring far distances to the back focal plane.

Diffraction has been lectured in the optics course and on the next pages we only repeat the most important relations to be discussed in this lab.

### 1. Single slit.

For a single slit with width  $a$  the angular distribution of diffracted intensity  $I_1$  will be given by:

$$I_1 = I_{01} \cdot \left[ \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right]^2 \cdot \text{where } \beta = \frac{2\pi a}{\lambda} \cdot \sin\theta \quad (1)$$

$\theta$  = diffraction angle

$\lambda$  = wavelength of the laser

$I_{01}$  = intensity for angle  $\theta = 0$

See also figure 1 for explanation of the remaining parameters.

The intensity distribution consists of a strong central maximum centered around  $\theta = 0$  and with much weaker side lobes on each side.

**The bi-maximum or the side lobes** will be located close to  $\sin(\beta/2) = \pm 1$ , that is

$\beta_N = (N + 1/2)\pi$  where  $N = 1, 2, 3, \dots$ . However, as equation 1 is divided by  $\beta/2$  these values will not be quite correct. A numerical solution gives:

$\beta_1 = 1.43\pi$ ,  $\beta_2 = 2.46\pi$ ,  $\beta_3 = 3.47\pi \dots$  Procedure: take the derivative of equation 1 – equal the result to zero – this lead to the general condition  $\tan x = x$  which you can solve on your calculator

**The Minima** in the pattern will be given by  $\sin(\beta/2) = 0$ , which leads to  $\beta_n = \pm n \cdot \pi$  where  $n = 1, 2, 3, \dots$ . By using the relation in equation 1 for  $\beta$  we find the minima as:

$$\sin\theta_{\min} = (\lambda/a) \cdot n$$

From equation 1 we see that the dimensions in the diffraction pattern is inversely proportional to the width of the slit  $a$ . A narrow slit results in broad, well separated maxima while a broad slit gives narrow, closely spaced maxima. Usually we know the wavelength with great accuracy and we can therefore determine the size of the slit (typically in the  $\mu\text{m}$  region) by measuring the dimensions in the diffraction pattern (typically in the  $\text{mm} / \text{cm}$  – region)

It can also be shown that an obstacle (wire, thread) with a cross-section the same size as the slit, will result in a pattern with the same periodic form as from the slit. (the slit opening and

its inverse obstacle are called complementary openings). We can therefore measure the diameter of very thin wires – this is used for industrial purposes).

### Two slits.



Figure 2.

The dimensions in the double slit is shown on figure 2 where we also show the transmission function of the double slit. Each of the slit openings will diffract a pattern given by equation 1. In the far field these two patterns will overlap and mutually interfere. (If the two slits are very thin, the central maximum of the diffraction will cover most of the observation plane and we can describe the pattern as pure interference – the famous Young’s double slit experiment)

Formally the intensity pattern resulting from two slits in a distance **d** is given as:

$$I_2 = I_{02} \cdot \left[ \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right]^2 \cdot \cos^2\left(\frac{\phi}{2}\right) \cdot \text{where } \phi = \frac{2\pi d}{\lambda} \cdot \sin \theta \quad (2)$$

where:

$I_{02}$  = the intensity in the direction  $\theta = 0$ .

Note that due to symmetry the amplitude of each slit is added to give twice the amplitude. However, we observe and record the intensity given by (amplitude)<sup>2</sup> and  $I_{02} = 4 I_{01}$

Equation (2) consist of the product of a **contribution from diffraction** (the first term) and a **contribution from interference** (the second term). The diffraction term is identical to what we found for a single slit.

The maxima of the interference term are all equally strong **before** they are multiplied by the diffraction term. Their angular directions are given by  $\phi_n = \pm m \cdot \pi$ . ( $m = 0, 1, 2, \dots$ ).

The interference maxima are called (grating) orders **where zero order goes straight ahead,  $\pm 1$  order is the first maxima on each side etc.** (NB! the zero order always follow geometrical

optics – in a transmission grating the direction of the zero order is given by refraction through the grating substrate, in a reflection grating the direction is given by the (mirror) reflection law)

Expressed by the direction  $\theta$ , the interference condition for maxima (orders) leads to the well known grating equation:

$$d \sin \theta = \pm m \cdot \lambda. \quad (m = 0, 1, 2, \dots). \quad (2a)$$

The minima in the interference term are of course midway between the maxima.

As the multiplying diffraction term has zeroes we experience in some cases that a zero in the diffraction term coincide with the direction for a grating order. This grating order will then disappear – so called missing order - and as the diffraction minima and the interference maxima are both periodic, we will find a periodic (multiple) disappearance of orders.

We find the condition for missing orders by eliminating the common term  $\sin \theta$  from the equations 1 and 2 which results in:

$$\frac{(\text{spalteavstand}) \ d}{(\text{spalteåpning}) \ a} = \frac{m \ (\text{gitterorden})}{n \ (\text{spalteminimum})}$$

If for example the slit separation  $d$  is 3 times larger than the width  $a$ , all  $n \cdot 3$  (grating) orders will be missing in the pattern.

### Multiple slits (grating).

Each of the slits diffracts the incoming light according to equation 1. All the  $N$  slits interfere which leads to interference maxima in the same directions as for the double slit. However, as the condition for interference is fulfilled for a great number of slit waves at the same time, we get sharper maxima and an increased peak intensity.

Mathematically the diffraction pattern from  $N$ -slits can be expressed as:

$$I_N = I_{01} \cdot \left[ \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right]^2 \cdot \left[ \frac{\sin\left(\frac{N}{2} \phi\right)}{\sin\left(\frac{\phi}{2}\right)} \right]^2 \cdot \text{where} \cdot \phi = \frac{2\pi d}{\lambda} \cdot \sin \theta \quad (3)$$

where as before  $I_{01}$  = intensity for  $\theta = 0$  for a single slit. By letting  $\theta \rightarrow 0$  in the second term of equation 3 we find that  $I_N = N^2 I_{01}$

Again, we see that the diffraction term from the single slit is multiplied by the term for the multiple interference. As the direction for the orders are the same regardless of the number of slits involved, the argument for missing orders will be unchanged from the double slit.

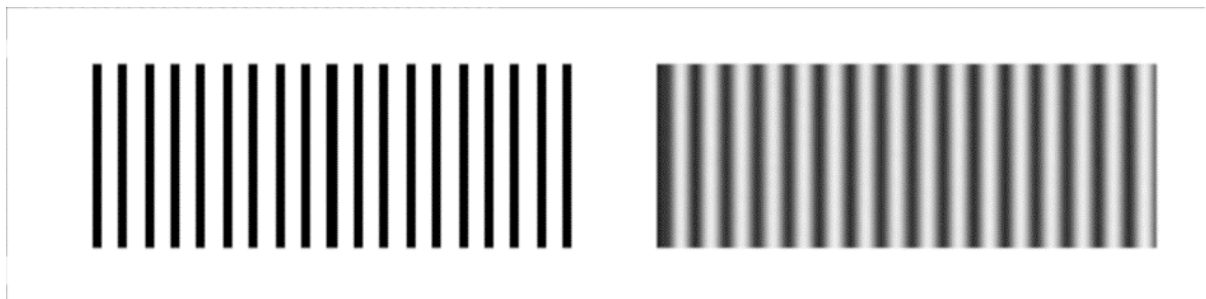
### **Simple gratings. Fourier description.**

#### **Sinusoidal grating.**

The simplest periodic structure is a sinusoidal grating where the transmission in one direction has a sinusoidal variation. An ideal sinus grating which we can vary between + and – values forms the basis for the two-dimensional Fourier Transform. In the far field such a grating will give two side orders representing + /- spatial frequency of the grating. Although it can be made, such a grating having positive and negative transmission is more the exception than the rule. For practical grating studies we have to work with gratings where the maximum transmission varies between 0 and 1. This means that we have added a constant value (DC) which will give a zero order at zero frequency. In the lectures the relation between dimensions in grating space and the far field will be discussed.

#### **Square wave grating.**

The square wave grating is just another name for the multiple slits described earlier. If we are using Fourier Transforms – the transmission of the grating can be described as a sum of sinusoidal gratings. Each +/- order is then represented by its corresponding grating. It all goes back to the standard description where a puls train (square wave function) can be described as a



sum of elementary sinus functions – also called overharmonics in acoustics.

Figure 3

Figure 3 show a square wave grating to the left and an (approximately) sinusoidal grating to the right. (Due to nonlinearities in the printing we cannot expect the sinus-grating to be pure..)

## Experiments.

The instrumental apparatus will be explained in detail in the lab. Roughly described, it consists of a standard diffraction set-up with laser illumination of the object and where the resulting far field is observed in the back focal plane of a lens. The patterns can be studied on a screen, but in the major part of the exercise we will record it by a photo detector covered with a very narrow slit. During measurement the detector is moved over the pattern by means of a so called step-motor which is controlled by the computer. The recording consists of a step – and repeat process which in our case provides 200 separated recordings of intensity and corresponding position over the field. By measuring several times and averaging we reduce the noise which comes mainly from the amplifier. Printouts of the results is obtained from a central laser printer.

### **1. Single slits**

Connect the control unit for the step motor, turn on the laser and the computer. Start the program FraunDiff. The program is reasonable self explaining, but unfortunately in Norwegian ( note to understand the menus there is a Norwegian – English Glossary at the end of this text) + that the teacher will go through the most important feature. The measurement range can be done over Default values 30 to 70 mm when the direction of the laser is adjusted to make the beam hit at about 50 mm referred to the scale on the detector carrier. You are however free to set different measurement ranges if you for example want study some features in more detail.

Print out of the recordings can be done by a central laser printer. Do not print out from the Data menu as that gives you only numbers that are not to exciting to look at.

**Use the object with two (widely) separated slits.**

**Record the diffraction pattern of each slit with two averaging runs (middling) Use the simulation menu to get the optimum coincidence between the recording and the theoretical function.**

**Make a print out of the results both in linear and logarithmic scale.**

**What is the advantage of a logarithmic scale?**

**Calculate the slit width from the distances between any defined minima on each side of the centre (pointing the cursor to a certain point on the recording provides the intensity and position of that point).**

**Compare the result with the slit width as given on the simulation print out.**

## **2. Multiple slits.**

The slits are here computer printed on a high contrast film and placed in a 35 mm slide holder. There are 1, 2, 3 and 10 slits in groups – all slits have the same width and separation  $d$ . Be sure that the laser beam is centered on the slit group before starting recording.

**Record and print out the diffraction patterns for the slit(s).**

Use a linear scale and 2 averaging runs. (Using 10 slits the light intensity will be so high that the A/D convertor saturates, but the distances recorded are still correct).

**Calculate the distance  $d$  between the slits by measuring the distance between the orders.**

**Find the relation between the intensities for the zero order for 1, 2 and 3 slits and compare with theoretical values as expressed by equation 3.**

**From the recordings and by observation of the patterns -which orders are missing?**

**What is then the width of one slit?**

**The recording of 3 slits shows clearly secondary maxima between the orders –what is the origin of these maxima?**

## **3. Thread object (inverse slit).**

The diffraction pattern of a thread or wire will have the same distance between the zeroes as a slit of the same width. The main difference is the wire pattern has a very strong zero order which is due to all the light passing the (thin) wire undiffracted. The recording can be made by the same equipment as above, the centre value will be very saturated, but using logarithmic scale it is possible to measure position of the minima.

**Record the pattern and determine the diameter of the wire by simulation.**

**(you can also measure the diameter of your own hair if you want – contact the supervisor for a special holder and tape.)**

## **4. Sinus grating.**

The gratings are placed in a holder – observe that the gratings are oriented correctly before you make a recording of the grating. (unfortunately the zero order saturates the A/D convertor for some grating so the modulation can not be determined),

**From the recording you can now determine:**

**The spatial frequency of the grating**

**The modulation of the grating**

**Are there any harmonics present?**

## 5. Spectral analysis with grating.

You first use the diffraction set up and the computer program

**Measure the grating constant  $d$  (the slit distance) by measuring the distance between the orders and using equation 2a**

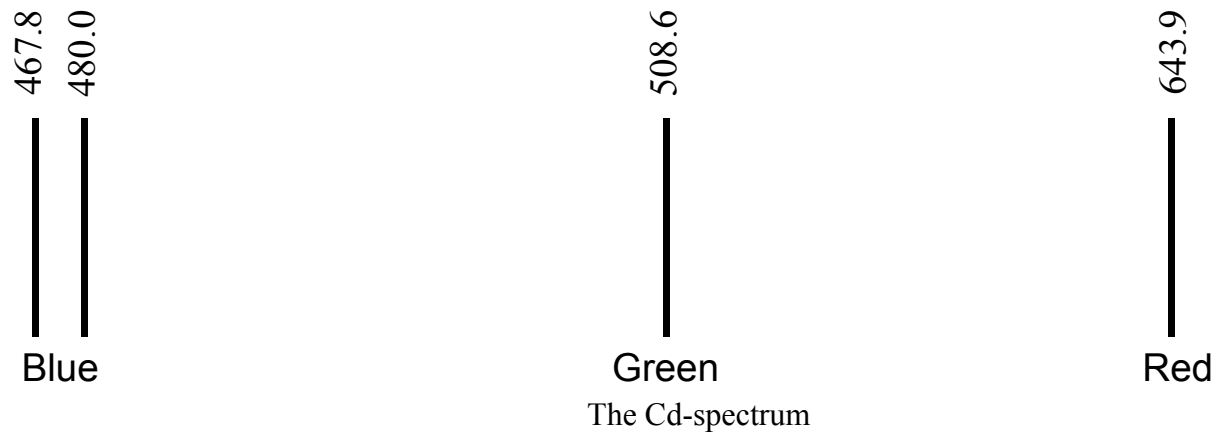
Place thereafter the grating in the holder on the spectrometer table – be sure that it is reasonable normal to the illumination from the collimator. The lamp is a Cd (Cadmium) lamp and its spectrum is shown below. In case you are not familiar with the use of a spectrometer, I will give a short introduction in the lab.

**What is the difference between the zero order spectrum and the higher order spectra you observe.**

**What are the first spectral orders where the spectra overlap?**

**Measure the angular deviation for one wavelength in the 5<sup>th</sup> order spectrum (or higher) to both sides and calculate  $d$  from equation 2a.**

the wavelength you choose is irrelevant, you just have to be sure that you can follow it over several orders. For my eyes the greenish line is best, with dark adapted eyes it can be followed up to at least 20<sup>th</sup> order, but this might differ from person to person depending on her/his spectral response of the eye. The prominent red line has a tendency to blend in with other lines in higher orders



**distance object - detector 1000 mm**

**Wavelength of laser  $\lambda = 635$  nm**

**Norwegian – English:**



**Graf** – Graph

Skriv ut – print out

Intensitetsakse -intensity scale (linear or logarithmic)

Linje – line ( på –on draws lines between points)

**Måling** – Measurements

Posisjonering – positioning (allows you to move detector in steps 10- 1 - 0.1mm)

Opptak – recording

Nytt opptak med middling – new recording with averaging

**Simulering** – simulation

Enkelt spalte – single slit

Notice also self explaining icons on top of screen. Pointing the cursor at a recording allows you to measure intensity and position accurately. Values are shown in the lower right corner.