Øving 2, Oppgave 1

Oppgave 1

$$(\vec\sigma\cdot\vec\pi)^2 = \vec\pi^2 + i\,\vec\sigma\cdot\underbrace{(\vec\pi\times\vec\pi)}_{\text{Operator!}}$$

$$i\vec\sigma\cdot(\vec p - q\vec A)\times(\vec p - q\vec A)\psi$$

$$= i\vec\sigma\left(\frac{\hbar}{i}\nabla - q\vec A\right)\times\left(\frac{\hbar}{i}\nabla - q\vec A\right)\psi$$

$$= i\vec\sigma\cdot\Big[\frac{\hbar^2}{i^2}\underbrace{\nabla\times\nabla}_{=0}\psi - \frac{\hbar q}{i}\nabla\times(\vec A\psi)$$

$$- \frac{\hbar q}{i}\vec A\times\nabla\psi$$

$$+ q^2\underbrace{\vec A\times\vec A}_{=0}\psi$$

$$= -i\,\vec\sigma\cdot\frac{\hbar q}{i}\Big(\big(\nabla\times\vec A\big)\psi - \vec A\times\nabla\psi + \vec A\times\nabla\psi\Big]$$

$$= -(\vec\sigma\cdot\vec B)\psi\,\hbar q$$ q.e.d

Oppgave 2 Dresselhaus

$$H = \frac{p^2}{2m} + \beta(\sigma_1 p_x - \sigma_2 p_y)$$

$$= \frac{p^2}{2m} + \beta\begin{pmatrix} 0 & p_x + ip_y \\ p_x - ip_y & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} \frac{p^2}{2m} & \beta p_+ \\ \beta p_- & \frac{p^2}{2m} \end{pmatrix}$$

$$H\Psi = E\Psi$$

$$\begin{pmatrix} \frac{p^2}{2m} - E & \beta p_+ \\ \beta p_- & \frac{p^2}{2m} - E \end{pmatrix} \Psi = 0$$

$$\left(\frac{p^2}{2m} - E\right)^2 = \beta^2 p_\perp^2 \quad ; \quad p_\perp^2 = p_x^2 + p_y^2$$

$$E = \frac{p^2}{2m} \pm \beta p_\perp$$

$$= \frac{1}{2m} p_z^2 + \frac{p_\perp^2}{2m} \pm \beta p_\perp$$

$$= \frac{p_z^2}{2m} + \frac{1}{2m}\left(p_\perp^2 \pm 2m\beta p_\perp\right)$$

$$= \frac{p_z^2}{2m} + \frac{1}{2m}\left[\left(p_\perp \pm \beta m\right)^2 - \beta^2 m^2\right]$$

Ian for $p_0 = \beta m$

$$E = \frac{p_z^2}{2m} + \frac{1}{2m}\left[(p_\perp + s\,p_D)^2 - p_D^2\right]$$

$s = \pm 1$

Set $p_z = 0$ $\therefore$ $\vec{p} = p_x \hat{i} + p_y \hat{j}$

$$\psi = N e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\vec{p} = \hbar \vec{k} \qquad \vec{k} = k_x \hat{i} + k_y \hat{j}$$

$$\begin{pmatrix} \frac{\hbar^2 k_\perp^2}{2m} - E & \beta \hbar k_+ \\ \beta \hbar k_- & \frac{\hbar^2 k_\perp^2}{2m} - E \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$$\frac{\hbar^2 k_\perp^2}{2m} - E = -s\,\beta \hbar k_\perp$$

$$-s\beta \hbar k_\perp a + \beta \hbar k_+ b = 0$$

$$a = s \frac{k_+}{k_\perp} b$$

$$\psi = N e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} s \frac{k_+}{k_\perp} \\ 1 \end{pmatrix}$$

$\vec{r} \to -\vec{r}$

$\vec{\sigma} \to -\vec{\sigma}$

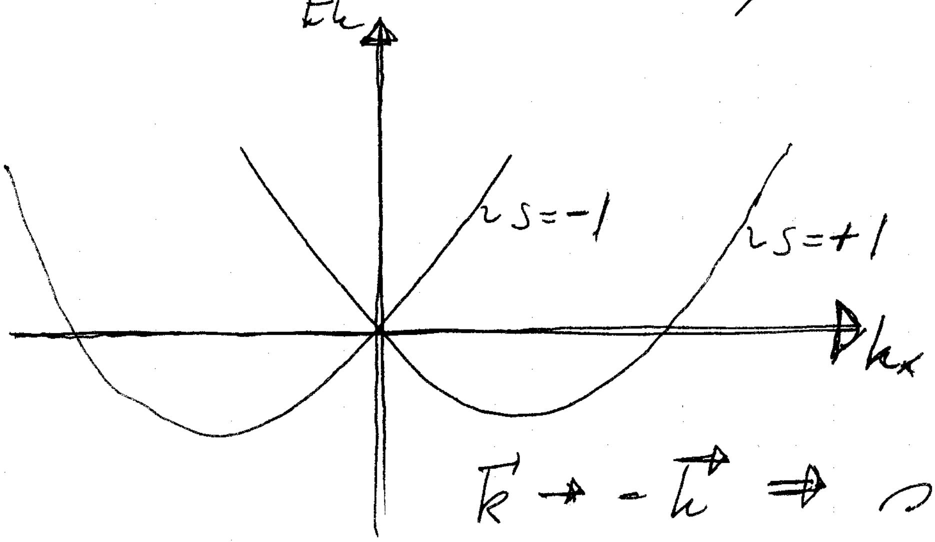
$\vec{p} \to -\vec{p}$

$$\frac{p^2}{2m} \to \frac{p^2}{2m}$$

$\cancel{\beta\, \vec{\sigma}\cdot\vec{p}_\perp}$

$$\beta(\sigma_1 p_x - \sigma_2 p_y) \to \beta(\sigma_1 p_x - \sigma_2 p_y)$$

$H \to H$ $\Rightarrow$: H har r-inversjons-symmetri.



$\vec{k} \to -\vec{k} \Rightarrow s \to -s$

$\vec{k}_+ \to -\vec{k}$

$\psi \to \psi$

ψ har r-inversjons-symmetri.

$$\Sigma = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}$$

$$\Sigma\psi = \frac{1}{|\vec{p}|} N e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} 0 & p_- \\ p_+ & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \frac{N e^{i\vec{k}\cdot\vec{r}}}{|\vec{p}|} \begin{pmatrix} p_- b \\ a p_+ \end{pmatrix}$$

$\neq$ const ψ_s

ψ_s <u>ikke</u> egentilstand til $\underline{\Sigma}$