Oppg. 1

Annet eksempel.

a) $$\left(c\alpha_r\left(p_r + i\beta\frac{K}{r}\right) + \beta(mc^2 + U) - E\right)\psi = 0$$

skalar-potensial!

U: kulesymm. skalar-pot. Kjernekraft!

Leder til radial-ligningene:

$$\left(\frac{\partial}{\partial r} + \frac{k}{r}\right)f + \left[\frac{mc^2}{\hbar c} - \frac{E}{\hbar c} + \frac{U(r)}{\hbar c}\right]g = 0$$

$$\left(\frac{\partial}{\partial r} - \frac{k}{r}\right)g + \left[\frac{mc^2}{\hbar} + \frac{E}{\hbar c} + \frac{U(r)}{\hbar c}\right]f = 0$$

Eneste forskjell fra ligningen for H-atomet! Fortegnet! For H-atomet hadde vi et minus-tegn her!

$$U = -\frac{A}{r}$$

Finn energinivåene.

Innfører $\alpha = \frac{A}{\hbar c}$

$$\nu = \sqrt{\frac{mc^2 - E}{mc^2 + E}}$$

$$\varkappa = \frac{1}{\hbar c}\sqrt{(mc^2)^2 - E^2}$$

$$\frac{1}{\hbar c}(mc^2 - E) = \frac{1}{\hbar c}\sqrt{(mc^2)^2 - E^2}\sqrt{\frac{mc^2 - E}{mc^2 + E}}$$

$$= \varkappa\nu$$

$$\frac{1}{\hbar c}(mc^2 + E) = \frac{\varkappa}{\nu}.$$

$$\left(\frac{\partial}{\partial r} + \frac{k}{r}\right) f + \left(\varkappa \nu - \frac{\alpha}{r}\right) g = 0$$

$$\left(\frac{\partial}{\partial r} - \frac{k}{r}\right) g + \left(\frac{\varkappa}{\nu} - \frac{\alpha}{r}\right) f = 0$$

$$\rho = \varkappa r \Rightarrow$$

$$\left(\frac{\partial}{\partial \rho} + \frac{k}{\rho}\right) f + \left(\nu - \frac{\alpha}{\rho}\right) g = 0$$

$$\left(\frac{\partial}{\partial \rho} - \frac{k}{\rho}\right) g + \left(\frac{1}{\nu} - \frac{\alpha}{\rho}\right) f = 0$$

$$f = e^{-\rho} F = e^{-\rho} \sum_\mu b_\mu \rho^{s+\mu}$$

$$g = e^{-\rho} G = e^{-\rho} \sum_\mu a_\mu \rho^{s+\mu}$$

Setter inn

$$-F + F' + \frac{k}{\rho} F + \nu G - \frac{\alpha}{\rho} G = 0$$

$$-G + G' - \frac{k}{\rho} G + \frac{1}{\nu} F - \frac{\alpha}{\rho} F = 0$$

$$\sum_\mu \Big\{ \left[(\mu + s + k) b_\mu - \alpha a_\mu\right] \rho^{s+\mu-1}$$

$$+ (\nu a_\mu - b_\mu) \Big\} \rho^{s+\mu} = 0$$

$$\sum_\mu \Big\{ \left[(\mu + s - k) a_\mu - \alpha b_\mu\right] \rho^{s+\mu-1} + \left(\frac{1}{\nu} b_\mu - a_\mu\right) \rho^{s+\mu}$$

$$(s+k)\,b_0 = \alpha\, a_0$$

$$(s-k)\,a_0 = \alpha\, b_0$$

$$(s-k)\,a_0 = \frac{\alpha^2}{s+k}\,a_0$$

$$s^2 - k^2 = \alpha^2$$

$$s^2 = k^2 + \alpha^2$$

$$\underline{s = \sqrt{k^2+\alpha^2}}$$

$$\text{I} \quad (s+\mu+k)\,b_\mu - \alpha\, a_\mu = b_{\mu-1} - \nu\, a_{\mu-1}$$

$$\text{II} \quad (s+\mu-k)\,a_\mu - \alpha\, b_\mu = a_{\mu-1} - \frac{1}{\nu}\, b_{\mu-1}$$

$$\text{I} + \nu\,\text{II} \Rightarrow$$

I: Samme som for H-atomet!
II: Eneste forskjell fra H-atomet fortegnet på leddet $\underline{\underline{\alpha b_\mu}}$!!

$$(s+\mu+k)\,b_\mu - \alpha\, a_\mu + \nu(s+\mu-k)\,a_\mu - \nu\alpha\, b_\mu$$

$$= b_{\mu-1} - \nu\, a_{\mu-1} + \nu\, a_{\mu-1} - b_{\mu-1} = 0$$

$$\left[s+\mu+k - \nu\alpha\right] b_\mu = \left[\alpha - \nu(s+\mu-k)\right] a_\mu$$

$$b_\mu = \frac{\left[\alpha - \nu(s+\mu-k)\right]}{s+\mu+k-\nu\alpha}\, a_\mu$$

$$b_{\mu-1} = \frac{\left[\alpha - \nu(s+\mu-1-k)\right]}{s+\mu-1+k-\nu\alpha}\, a_{\mu-1}$$

Innsatt i I:

$$(s+\mu+h)\frac{[\alpha-\nu(s+\mu-h)]}{s+\mu+h-\nu\alpha}\,a_\mu - \alpha a_\mu$$

$$= \frac{[\alpha-\nu(s+\mu-1-h)]}{s+\mu-1+h-\nu\alpha}\,a_{\mu-1} - \nu a_{\mu-1}$$

$$\cancel{\frac{a_\mu}{a_{\mu-1}} = \frac{\alpha - \frac{[\alpha-\nu(s+\mu-h)]}{s+\mu+h-\nu\alpha}(s+\mu+h)}{\nu - \frac{\alpha-\nu(s+\mu-1-h)}{s+\mu-1+h-\nu\alpha}}}$$

$$\cancel{\mu \to \infty \Rightarrow \frac{a_\mu}{a_{\mu-1}} \to 0}$$

$$\frac{a_\mu}{a_{\mu-1}} = \frac{\nu - \frac{\alpha-\nu(s+\mu-1-h)}{s+\mu-1+h-\nu\alpha}}{\alpha-(s+\mu+h)\left(\frac{\alpha-\nu(s+\mu-h)}{s+\mu+h-\nu\alpha}\right)}$$

$\mu \to \infty \Rightarrow$

$$\frac{a_\mu}{a_{\mu-1}} = \frac{\nu+\nu}{-\mu(-\nu)} = \frac{2}{\mu} \Rightarrow$$

relation for $e^{2z} \Rightarrow f(z) \sim e^{2z}$

For å få det. egf.:

Rekken må bryte av

$$a_{\mu_0+1} = 0$$

$$a_{\mu_0} = 0$$

$$\nu - \frac{\alpha - \nu(s+\mu_0 - k)}{s+\mu_0+k-\nu\alpha} = 0$$

$$\alpha - \nu(s+\mu_0-k) = \nu(s+\mu_0) + \nu k - \nu^2\alpha$$

$$\alpha - \nu(s+\mu_0) + \nu k = \nu(s+\mu_0) + \nu k - \nu^2 \alpha$$

$$2\nu(s+\mu_0) = \alpha(1+\nu^2)$$

NB!! I H-atomet hadde vi minus-tegn her!

$$2(s+\mu_0) = \alpha\left(\nu + \frac{1}{\nu}\right)$$

$$= \alpha\left(\sqrt{\frac{mc^2-E}{mc^2+E}} + \sqrt{\frac{mc^2+E}{mc^2-E}}\right)$$

$$= \alpha\left(\frac{mc^2-E+mc^2+E}{\sqrt{(mc^2)^2-E^2}}\right)$$

$$2(s+\mu_0) = \frac{2\alpha mc^2}{\sqrt{(mc^2)^2-E^2}}$$

I H-atomet hadde vi E istedet for mc^2 i telleren!

$$(mc^2)^2 - E^2 = \frac{(\alpha\, mc^2)^2}{(s+n_r)^2}$$

$$E^2 = (mc^2)^2 \left\{ 1 - \frac{\alpha^2}{(s+n_r)^2} \right\}$$

$$E = \pm\, mc^2 \sqrt{1 - \frac{\alpha^2}{(s+n_r)^2}}$$

$$= \pm\, mc^2 \sqrt{1 - \frac{\alpha^2}{\left(n_r + \sqrt{k^2+\alpha^2}\right)^2}}$$

$$= \pm\, mc^2 \sqrt{1 - \frac{\alpha^2}{\left(n_r + \sqrt{(j+\frac{1}{2})^2+\alpha^2}\right)^2}}$$

$$= \pm\, mc^2 \sqrt{1 - \frac{\alpha^2}{\left(n-(j+\frac{1}{2}) + \sqrt{(j+\frac{1}{2})^2+\alpha^2}\right)^2}}$$

$\alpha = \frac{A}{\hbar c}$

Forskjell fra relativistisk hydrogen-atom:

Feltet her er *ikke* elektrisk kraft. Et inngår på annen måte i Dirac-ligningen enn hva det elektrost. pot. gjør

Oppg. 1b

<u>Kjerne-kraften</u>

For H-atomet hadde vi spektret

$$E = \pm \frac{mc^2}{\sqrt{1 + \frac{Z^2\alpha^2}{\left(n-(j+\frac{1}{2}) + \sqrt{(j+\frac{1}{2})^2 + Z^2\alpha^2}\right)^2}}}$$

Med skalarfeltet ... fikk vi istedet

$$E = \pm mc^2 \sqrt{1 - \frac{Z^2\alpha^2}{\left(n-(j+\frac{1}{2}) + \sqrt{(j+\frac{1}{2})^2 + Z^2\alpha^2}\right)^2}}$$

I H-atomet: „Katastrofe" hvis $Z\alpha = j + \frac{1}{2}$

I vårt andre tilfelle: „Katastrofe" dersom

$$\underbrace{n-(j+\tfrac{1}{2})}_{x} + \sqrt{\underbrace{(j+\tfrac{1}{2})^2}_{a} + Z^2\alpha^2} = Z\alpha$$

$$a^2 + Z^2\alpha^2 = (Z\alpha - x)^2 = Z^2\alpha^2 - 2Z\alpha x + x^2$$

$$2Z\alpha x = x^2 - a^2$$

$$2Z\alpha(n-(j+\tfrac{1}{2})) = (j+\tfrac{1}{2})^2 - 2n(j+\tfrac{1}{2}) + n^2 - (j+\tfrac{1}{2})^2$$

$$2Z\alpha\left[n-(j+\tfrac{1}{2})\right] = n\left(n - 2(j+\tfrac{1}{2})\right)$$

$$Z\alpha = \frac{n}{2}\left[\frac{n-2(j+1/2)}{n-(j+1/2)}\right]$$

Oppg. 1c Dirac ligning i polar-koordinater med elektromagnetisk felt:

$$A_\mu = \left(\frac{\varphi}{c}, \vec{A}\right)$$

$$[c\,\vec{\alpha}\cdot(\vec{p} - e\vec{A}) + \beta mc^2 + e\varphi - E]\,\psi = 0$$

$$[c\,\vec{\alpha}\cdot(\vec{p} - e\vec{A}) + \beta mc^2 + V(r) - E]\,\psi = 0$$

$$\vec{\pi} = \vec{p} - e\vec{A}$$

$$\vec{\alpha}\cdot\vec{\pi} = \alpha_r^2\,\vec{\alpha}\cdot\vec{\pi}$$

$$= \frac{\alpha_r}{r}\,\vec{\alpha}\cdot\vec{r}\;\vec{\alpha}\cdot\vec{\pi}$$

$$= \frac{\alpha_r}{r}\left(\vec{r}\cdot\vec{\pi} + i\vec{\Sigma}\cdot\vec{r}\times\vec{\pi}\right)$$

$$= \frac{\alpha_r}{r}\left\{\vec{r}\cdot\vec{p} - e\,\vec{r}\cdot\vec{A} + i\vec{\Sigma}\cdot\vec{r}\times(\vec{p} - e\vec{A})\right\}$$

$$= \frac{\alpha_r}{r}\left(\vec{r}\cdot\vec{p} - e\,\vec{r}\cdot\vec{A} + i\vec{\Sigma}\cdot\vec{L} - ie\,\vec{\Sigma}\cdot\vec{r}\times\vec{A}\right)$$

Spesial-tilfelle: $\vec{A} = A(r)\,\hat{r}$

$$\Rightarrow \vec{r}\times\vec{A} = 0$$

$$\alpha\cdot\overline{\pi} = \frac{\alpha_r}{r}\left(r p_r + i\hbar - c r A + i \overline{\Sigma}\cdot\overline{L}\right)$$

$$= \alpha_r \left(p_r - cA + i\beta\frac{K}{r}\right)$$

<u>Innsatt i Dirac-ligning.</u>

$$\left[c\alpha_r\left(p_r - eA + i\beta\frac{K}{r}\right) + \beta mc^2 + V - E\right]\psi = 0$$

$$\psi = \begin{pmatrix}\psi_a\\ \psi_b\end{pmatrix} = \begin{pmatrix}\frac{i\,G(r)}{r}\mathcal{Y}_{j\ell\frac{1}{2}}^{m}\\ \frac{F(r)}{r}\mathcal{Y}_{j\ell'\frac{1}{2}}\end{pmatrix}$$

$$\Big\{c\begin{pmatrix}0 & \sigma_r\\ \sigma_r & 0\end{pmatrix}\begin{pmatrix}p_r - eA & \\ & p_r - eA\end{pmatrix} + ic\begin{pmatrix}0 & \sigma_r\\ \sigma_r & 0\end{pmatrix}\begin{pmatrix}\frac{\hbar k}{r} & \\ & -\frac{\hbar k}{r}\end{pmatrix}$$

$$+ \begin{pmatrix}mc^2 + V - E & \\ & -mc^2 + V - E\end{pmatrix}\Big\}\begin{pmatrix}\psi_a\\ \psi_b\end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & c\left(\sigma_r\left[(p_r - eA) - i\frac{\hbar k}{r}\right]\right) \\ c\left(\sigma_r\left[(p_r - eA) + \frac{i\hbar k}{r}\right]\right) & 0 \end{pmatrix}\begin{pmatrix}\psi_a \\ \psi_b\end{pmatrix}$$

$$+ \begin{pmatrix} mc^2 + V - E & 0 \\ 0 & V - (mc^2 + E)\end{pmatrix}\begin{pmatrix}\psi_a \\ \psi_b\end{pmatrix} = 0$$

$$(mc^2 + V - E)\psi_a + c\,\sigma_r\left[(p_r - eA) - i\frac{\hbar k}{r}\right]\psi_b = 0$$

$$c\,\sigma_r\left[(p_r - eA) + \frac{i\hbar k}{r}\right]\psi_a + \left[V - (mc^2 + E)\right]\psi_b = 0$$

$$(mc^2 + V - E)\frac{iG(r)}{r} - c\left(p_r - eA - \frac{i\hbar k}{r}\right)\frac{F(r)}{r} = 0$$

$$-\cancel{i}\,c\left(p_r - eA + \frac{i\hbar k}{r}\right)\frac{iG(r)}{r} + \left[V - (mc^2 + E)\right]\frac{F(r)}{r} = 0$$

$$i\hbar c\left[\frac{1}{r}\frac{\partial F}{\partial r} - \left(\frac{ieA}{\hbar} + \frac{k}{r}\right)F\right] + \frac{1}{r}(mc^2 + V - E)G = 0$$

$$\underline{\frac{1}{r}\frac{\partial F}{\partial r} - \left(\frac{ieA}{\hbar} - \frac{k}{r}\right)F + \frac{1}{r}\frac{(mc^2 + V - E)}{\hbar c}G = 0}$$

$$(-ic)(-i\hbar)\left[\frac{1}{r}\frac{\partial G}{\partial r} - \left(\frac{ie}{\hbar}A + \frac{\hbar k}{r}\right)G\right] + \left(V - mc^2 + E\right)\frac{F}{r}$$

$$\underline{\frac{1}{r}\frac{\partial G}{\partial r} - \left(\frac{ieA}{\hbar} + \frac{k}{r}\right)G - \left[V - (mc^2 + E)\right]_{\hbar c}\frac{F}{r} = 0}$$

Spezifische formen

på $A(r)$: $A(r) = \frac{D}{r}$

$$\Rightarrow \quad \frac{ieA}{\hbar} = \frac{ieD}{\hbar}\frac{1}{r} = \underline{\frac{\varepsilon}{r}}.$$

$$V = -\frac{B}{r} \Rightarrow \frac{V}{\hbar c} = -\frac{B}{\hbar c}\frac{1}{r} = -\frac{\alpha}{r}$$

$$\frac{1}{r}\frac{\partial F}{\partial r} - \left(\frac{\varepsilon}{r} - \frac{k}{r}\right)\frac{F}{r} + \frac{1}{r}\left(\frac{mc^2 - E}{\hbar} - \frac{\alpha}{r}\right)G = 0$$

$$\frac{1}{r}\frac{\partial G}{\partial r} - \left(\frac{\varepsilon}{r} + \frac{k}{r}\right)\frac{G}{r} + \frac{1}{r}\left(\frac{mc^2 + E}{\hbar} + \frac{\alpha}{r}\right)F = 0$$

$$\varepsilon - k = \varepsilon_1$$

$$\varepsilon + k = \varepsilon_2 \quad ; \quad \underline{k = \omega\left(j + \tfrac{1}{2}\right)}.$$

$$\varkappa = \frac{1}{\hbar c}\sqrt{(mc^2)^2 - E^2}$$

$$\nu = \sqrt{\frac{mc^2 - E}{mc^2 + E}}$$

$$\frac{mc^2 - E}{\hbar c} = \frac{1}{\hbar c}\sqrt{}\,\nu = \underline{\varkappa\nu}$$

$$\frac{mc^2 + E}{\hbar c} = \underline{\frac{\varkappa}{\nu}}.$$

$$\frac{\partial F}{\partial r} - \frac{\varepsilon_1}{r} F + \left(\varkappa \nu - \frac{\alpha}{r}\right) G = 0$$

$$\frac{\partial G}{\partial r} - \frac{\varepsilon_2}{r} G + \left(\frac{\varkappa}{\nu} + \frac{\alpha}{r}\right) F = 0$$

$$\rho = \varkappa r$$

$$\frac{\partial F}{\partial \rho} - \frac{\varepsilon_1}{\rho} F + \left(\nu - \frac{\alpha}{\rho}\right) G = 0$$

$$\frac{\partial G}{\partial \rho} - \frac{\varepsilon_2}{\rho} G + \left(\frac{1}{\nu} + \frac{\alpha}{\rho}\right) F = 0$$

$$F = e^{-\rho} f = e^{-\rho} \sum_\mu b_\mu \rho^{s+\mu}$$

$$G = e^{-\rho} g = e^{-\rho} \sum_\mu a_\mu \rho^{s+\mu}$$

$$-f + f' - \frac{\varepsilon_1}{\rho} f + \left(\nu - \frac{\alpha}{\rho}\right) g = 0$$

$$-g + g' - \frac{\varepsilon_2}{\rho} g + \left(\frac{1}{\nu} - \frac{\alpha}{\rho}\right) f = 0$$

$$\sum_\mu \left\{ \left[(\mu + s - \varepsilon_1) b_\mu - \alpha a_\mu \right] \rho^{s+\mu-1} + (\nu a_\mu - b_\mu) \rho^{s+\mu} \right\}$$

$$\sum_\mu \left[\left((\mu + s - \varepsilon_2) a_\mu + \alpha b_\mu \right) \rho^{s+\mu-1} + \left(\frac{b_\mu}{\nu} - a_\mu \right) \rho^{s+\mu} \right] = 0$$

Indiakot (indikator)

$$(s-\varepsilon_1)\, b_0 = \alpha a_0$$

$$(s-\varepsilon_2)\, a_0 = -\alpha b_0$$

$$(s-\varepsilon_2)\, a_0 = \frac{-\alpha^2}{s-\varepsilon_1}\, a_0$$

$$(s-\varepsilon_2)(s-\varepsilon_1) = -\alpha^2$$

$$s^2 - (\varepsilon_1+\varepsilon_2)\, s + (\varepsilon_1\varepsilon_2 + \alpha^2) = 0$$

$$s = \frac{\varepsilon_1+\varepsilon_2 \pm \sqrt{(\varepsilon_1+\varepsilon_2)^2 - 4(\varepsilon_1\varepsilon_2+\alpha^2)}}{2}$$

$$= \frac{\varepsilon_1+\varepsilon_2 \pm \sqrt{(\varepsilon_1-\varepsilon_2)^2 + 4\alpha^2}}{2}$$

$$\frac{2\varepsilon \pm \cancel{4}\sqrt{(k^2+\alpha^2)}}{2}$$

$$= \varepsilon \pm \sqrt{k^2+\alpha^2}$$

$$\varepsilon = \frac{i\varepsilon 0}{\hbar} \quad ; \quad \alpha = \frac{\beta}{\hbar c}$$

$$\mathrm{I} \quad (\mu+\vartheta-\varepsilon_1)b_\mu - \alpha a_\mu + \nu a_{\mu-1} - b_{\mu-1} = 0$$

$$\mathrm{II} \quad (\mu+\vartheta-\varepsilon_2)a_\mu + \alpha b_\mu + \frac{1}{\nu} b_{\mu-1} - a_{\mu-1} = 0$$

$\mathrm{I} + \nu\,\mathrm{II}$:

$$(\mu+\vartheta-\varepsilon_1)b_\mu - \alpha a_\mu + \nu a_{\mu-1} - b_{\mu-1}$$

$$+ \nu(\mu+\vartheta-\varepsilon_2)a_\mu + \nu\alpha b_\mu + b_{\mu-1} - \nu a_{\mu-1} = 0$$

$$(\mu+\vartheta-\varepsilon_1+\nu\alpha)b_\mu = [\alpha - \nu(\mu+\vartheta-\varepsilon_2)]a_\mu$$

$$\underline{b_\mu = \frac{\alpha-\nu(\mu+\vartheta-\varepsilon_2)}{\mu+\vartheta-\varepsilon_1+\nu\alpha}\, a_\mu}$$

$$\underline{b_{\mu-1} = \frac{\alpha-\nu(\mu-1+\vartheta-\varepsilon_2)}{\vartheta+\mu-1-\varepsilon_1+\nu\alpha}\, a_{\mu-1}}$$

$$\left[(\mu+\vartheta-\varepsilon_1)\frac{\alpha-\nu(\mu+\vartheta-\varepsilon_2)}{\mu+\vartheta-\varepsilon_1+\nu\alpha} - \alpha\right]a_\mu$$

$$= \left[\frac{\alpha-\nu(\mu-1+\vartheta-\varepsilon_2)}{\vartheta+\mu-1-\varepsilon_1+\nu\alpha} - \nu\right]a_{\mu-1}$$

$$\frac{a_\mu}{a_{\mu-1}} = \cfrac{\nu - \dfrac{\alpha - \nu(\mu - 1 + s - \varepsilon_2)}{s + \mu - 1 - \varepsilon_1 + \nu\alpha}}{\alpha - (\mu + s - \varepsilon_1)\dfrac{\alpha - \nu(\mu + s - \varepsilon_2)}{\mu + s - \varepsilon_1 + \nu\alpha}}$$

$$\mu \to \infty : \quad \frac{a_\mu}{a_{\mu-1}} \to \frac{?}{\mu} \to e^{2\rho}$$

Må bryte av.

$$\nu - \frac{\alpha - \nu(\mu - 1 + s - \varepsilon_2)}{s + \mu - 1 - \varepsilon_1 + \nu\alpha} = 0$$

$$\alpha \div \nu(\mu_0 \cancel{-1} + s \div \varepsilon_2) = \nu(s + \mu_0 - \varepsilon_1 + \nu\alpha)$$

$$\alpha - \nu(s + \mu_0) + \nu\varepsilon_2 = \nu(s + \mu_0) - \nu\varepsilon_1 + \nu^2\alpha$$

$$2\nu(s + \mu_0) = \nu(\varepsilon_1 + \varepsilon_2) + \alpha(1 \div \nu^2)$$

$$2(s + \mu_0) = 2\varepsilon + \alpha\left(\frac{1}{\nu} \div \nu\right)$$

$$= 2\varepsilon + \alpha\left(\sqrt{\frac{mc^2 + E}{mc^2 - E}} - \sqrt{\frac{mc^2 - E}{mc^2 + E}}\right)$$

$$= 2\varepsilon + \alpha\frac{2E}{\sqrt{(mc^2)^2 - E^2}}$$

$$E = \pm mc^2 \sqrt{1 - \frac{\alpha^2}{(s+\mu_0)^2}}$$

$$= \pm mc^2 \left\{ 1 - \frac{\alpha^2}{\left(\mu_0 + \sqrt{k^2+\alpha^2}\right)^2} \right\}^{\frac{1}{2}}$$

$$2(s+\mu_0) = 2\mathcal{E} + \alpha \frac{2E}{\sqrt{(mc^2)^2 - E^2}}$$

$$s + \mu_0 - \mathcal{E} = \frac{\alpha E}{(mc^2)^2 - E^2}$$

$$(s+\mu_0-\mathcal{E})^2 = \frac{\alpha^2 E^2}{(mc^2)^2 - E^2}$$

$$(s+\mu_0-\mathcal{E})^2 \left[(mc^2)^2 - E^2\right] = \alpha^2 E^2$$

$$(mc^2)^2 - E^2 = E^2 \frac{\alpha^2}{(s+\mu_0-\mathcal{E})^2}$$

$$E^2 \left[1 + \frac{\alpha^2}{(s+\mu_0-\mathcal{E})^2}\right] = (mc^2)^2$$

$$E = \pm \frac{mc^2}{\sqrt{1 + \dfrac{\alpha^2}{(s + \mu_0 - \varepsilon)^2}}}$$

$$s = \varepsilon + \sqrt{k^2 - \alpha^2}$$

$$s - \varepsilon = \sqrt{k^2 - \alpha^2}$$

$$= \sqrt{\left(j + \tfrac{1}{2}\right)^2 - \alpha^2}$$

$$\mu_0 = n - \left(j + \tfrac{1}{2}\right)$$

$$E = \pm \frac{mc^2}{\sqrt{1 + \dfrac{\alpha^2}{\left(\left(n - \left(j + \tfrac{1}{2}\right)\right) + \sqrt{\left(j + \tfrac{1}{2}\right)^2 - \alpha^2}\right)^2}}}$$

Energi-nivåene til det elektrost. pot. <u>ikke</u> påvirket av vårt spesielle vektorpot.: $\underline{\underline{\vec{A} = \frac{D}{r}\hat{r}}}$