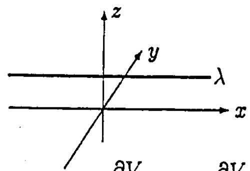
# Øving 2- Løsningsforslag

**Problem 3.2**

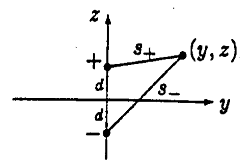
A stable equilibrium is a point of local minimum in the potential energy. Here the potential energy is $qV$. But we know that Laplace's equation allows no local minima for $V$. What *looks* like a minimum, in the figure, must in fact be a saddle point, and the box "leaks" through the center of each face.

**Problem 3.9**

(a) Image problem: $\lambda$ above, $-\lambda$ below. Potential was found in Prob. 2.47:




$$V(y,z) = \frac{2\lambda}{4\pi\epsilon_0}\ln(s_-/s_+) = \frac{\lambda}{4\pi\epsilon_0}\ln(s_-^2/s_+^2)$$
$$= \boxed{\frac{\lambda}{4\pi\epsilon_0}\ln\left\{\frac{y^2+(z+d)^2}{y^2+(z-d)^2}\right\}}$$




(b) $\sigma = -\epsilon_0\frac{\partial V}{\partial n}$. Here $\frac{\partial V}{\partial n} = \frac{\partial V}{\partial z}$, evaluated at $z = 0$.

$$\sigma(y) = -\epsilon_0\frac{\lambda}{4\pi\epsilon_0}\left\{\frac{1}{y^2+(z+d)^2}2(z+d) - \frac{1}{y^2+(z-d)^2}2(z-d)\right\}\Bigg|_{z=0}$$
$$= -\frac{2\lambda}{4\pi}\left\{\frac{d}{y^2+d^2} - \frac{-d}{y^2+d^2}\right\} = \boxed{-\frac{\lambda d}{\pi(y^2+d^2)}}.$$

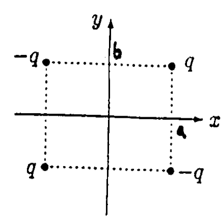
*Check:* Total charge induced on a strip of width $l$ parallel to the $y$ axis:

$$q_{\text{ind}} = -\frac{l\lambda d}{\pi}\int_{-\infty}^{\infty}\frac{1}{y^2+d^2}\,dy = -\frac{l\lambda d}{\pi}\left[\frac{1}{d}\tan^{-1}\left(\frac{y}{d}\right)\right]\Bigg|_{-\infty}^{\infty} = -\frac{l\lambda d}{\pi}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]$$
$$= -\lambda l. \text{ Therefore } \lambda_{\text{ind}} = -\lambda, \text{ as it should be.}$$

**Problem 3.10**

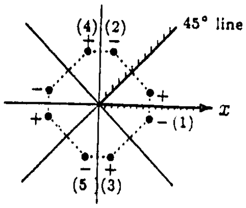
The image configuration is as shown.

$$\boxed{V(x,y) = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{\sqrt{(x-a)^2+(y-b)^2+z^2}} + \frac{1}{\sqrt{(x+a)^2+(y+b)^2+z^2}} - \frac{1}{\sqrt{(x+a)^2+(y-b)^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+(y+b)^2+z^2}}\right\}.}$$



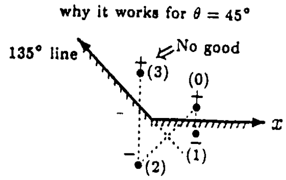

For this to work, $\boxed{\theta \text{ must be an integer divisor of } 180°.}$ Thus 180°, 90°, 60°, 45°, etc., are OK, but no others. It works for 45°, say, with the charges as shown.

(Note the strategy: to make the $x$ axis an equipotential ($V = 0$), you place the image charge (1) in the reflection point. To make the 45° line an equipotential, you place charge (2) at the image point. But that screws up the $x$ axis, so you must now insert image (3) to balance (2). Moreover, to make the 45° line $V = 0$ you also need (4), to balance (1). But now, to restore the $x$ axis to $V = 0$ you need (5) to balance (4), and so on.




why it works for $\theta = 45°$

The reason this doesn't work for *arbitrary* angles is that you are eventually forced to place an image charge *within the original region of interest*, and that's not allowed—*all* images must go *outside* the region, or you're no longer dealing with the same problem at all.)




why it *doesn't* work for $\theta = 135°$

Force on $q$ (Coulomb's law): $\mathbf{F} = -\frac{q^2}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{x}}}{(2a)^2} + \frac{\hat{\mathbf{y}}}{(2b)^2} - \frac{2a\hat{\mathbf{x}}+2b\hat{\mathbf{y}}}{\left((2a)^2+(2b)^2\right)^{3/2}}\right]$

Energy: $W = \int_{(\infty)}^{(a,b)}\mathbf{F}\cdot d\mathbf{l} = -\frac{q^2}{16\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{b}-\frac{1}{\sqrt{a^2+b^2}}\right]$, which is 1/4 of the electrostatic energy of four *real* charges. In this case the field fills only 1/4 of total space.