SIF 4060 Elektromagnetisk teori

Øving 3- Løsningsforslag

Problem 3.30

(a) (i) $Q = \boxed{2q,}$ (ii) $\mathbf{p} = \boxed{3qa\,\hat{\mathbf{z}},}$ (iii) $V \cong \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}\right] = \boxed{\frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{3qa\cos\theta}{r^2}\right]}.$

(b) (i) $Q = \boxed{2q,}$ (ii) $\mathbf{p} = \boxed{qa\,\hat{\mathbf{z}},}$ (iii) $V \cong \boxed{\frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{qa\cos\theta}{r^2}\right]}.$

(c) (i) $Q = \boxed{2q,}$ (ii) $\mathbf{p} = \boxed{3qa\,\hat{\mathbf{y}},}$ (iii) $V \cong \boxed{\frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{3qa\sin\theta\sin\phi}{r^2}\right]}$ (from Eq. 1.64, $\hat{\mathbf{y}}\cdot\hat{\mathbf{r}} = \sin\theta\sin\phi$).

Problem 3.31

(a) This point is at $r = a$, $\theta = \frac{\pi}{2}$, $\phi = 0$, so $\mathbf{E} = \frac{p}{4\pi\epsilon_0 a^3}\hat{\boldsymbol{\theta}} = \frac{p}{4\pi\epsilon_0 a^3}(-\hat{\mathbf{z}})$; $\mathbf{F} = q\mathbf{E} = \boxed{-\frac{pq}{4\pi\epsilon_0 a^3}\,\hat{\mathbf{z}}.}$

(b) Here $r = a$, $\theta = 0$, so $\mathbf{E} = \frac{p}{4\pi\epsilon_0 a^3}(2\hat{\mathbf{r}}) = \frac{2p}{4\pi\epsilon_0 a^3}\,\hat{\mathbf{z}}.$ $\boxed{\mathbf{F} = \frac{2pq}{4\pi\epsilon_0 a^3}\,\hat{\mathbf{z}}.}$

(c) $V = q\,[V(0,0,a) - V(a,0,0)] = \frac{qp}{4\pi\epsilon_0 a^2}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \boxed{\frac{pq}{4\pi\epsilon_0 a^2}}.$

Problem 3.32

$Q = -q$, so $V_{\text{mono}} = \frac{1}{4\pi\epsilon_0}\frac{-q}{r}$; $\mathbf{p} = qa\,\hat{\mathbf{z}}$, so $V_{\text{dip}} = \frac{1}{4\pi\epsilon_0}\frac{qa\cos\theta}{r^2}$. Therefore

$$\boxed{V(r,\theta) \cong \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r} + \frac{a\cos\theta}{r^2}\right).} \quad \boxed{\mathbf{E}(r,\theta) \cong \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r^2}\,\hat{\mathbf{r}} + \frac{a}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)\right].}$$

Problem 3.33

$\mathbf{p} = (\mathbf{p}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} + (\mathbf{p}\cdot\hat{\boldsymbol{\theta}})\,\hat{\boldsymbol{\theta}} = p\cos\theta\,\hat{\mathbf{r}} - p\sin\theta\,\hat{\boldsymbol{\theta}}$ (Fig. 3.36). So $3(\mathbf{p}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} - \mathbf{p} = 3p\cos\theta\,\hat{\mathbf{r}} - p\cos\theta\,\hat{\mathbf{r}} + p\sin\theta\,\hat{\boldsymbol{\theta}} = 2p\cos\theta\,\hat{\mathbf{r}} + p\sin\theta\,\hat{\boldsymbol{\theta}}$. So Eq. 3.104 ≡ Eq. 3.103. ✓