Øving 4- Løsningsforslag

Problem 4.14

Total charge on the dielectric is $Q_{\text{tot}} = \oint_S \sigma_b\, da + \int_V \rho_b\, d\tau = \oint_S \mathbf{P}\cdot d\mathbf{a} - \int_V \nabla\cdot\mathbf{P}\, d\tau$. But the divergence theorem says $\oint_S \mathbf{P}\cdot d\mathbf{a} = \int_V \nabla\cdot\mathbf{P}\, d\tau$, so $Q_{\text{enc}} = 0$. qed

Problem 4.21

Let Q be the charge on a length ℓ of the inner conductor.

$$\oint \mathbf{D}\cdot d\mathbf{a} = D2\pi s\ell = Q \Rightarrow D = \frac{Q}{2\pi s\ell};\quad E = \frac{Q}{2\pi\epsilon_0 s\ell}\ (a<s<b),\quad E = \frac{Q}{2\pi\epsilon s\ell}\ (b<r<c).$$

$$V = -\int_c^a \mathbf{E}\cdot d\mathbf{l} = \int_a^b \left(\frac{Q}{2\pi\epsilon_0\ell}\right)\frac{ds}{s} + \int_b^c \left(\frac{Q}{2\pi\epsilon\ell}\right)\frac{ds}{s} = \frac{Q}{2\pi\epsilon_0\ell}\left[\ln\left(\frac{b}{a}\right) + \frac{\epsilon_0}{\epsilon}\ln\left(\frac{c}{b}\right)\right].$$

$$\frac{C}{\ell} = \frac{Q}{V\ell} = \boxed{\frac{2\pi\epsilon_0}{\ln(b/a) + (1/\epsilon_r)\ln(c/b)}}.$$

Problem 4.33

$\mathbf{E}^{\parallel}$ is continuous (Eq. 4.29); $D_\perp$ is continuous (Eq. 4.26, with $\sigma_f = 0$). So $E_{x_1} = E_{x_2}$, $D_{y_1} = D_{y_2} \Rightarrow \epsilon_1 E_{y_1} = \epsilon_2 E_{y_2}$, and hence

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{E_{x_2}/E_{y_2}}{E_{x_1}/E_{y_1}} = \frac{E_{y_1}}{E_{y_2}} = \frac{\epsilon_2}{\epsilon_1}.\quad \text{qed}$$

If 1 is air and 2 is dielectric, $\tan\theta_2/\tan\theta_1 = \epsilon_2/\epsilon_0 > 1$, and the field lines bend *away* from the normal. This is the opposite of light rays, so a convex "lens" would *defocus* the field lines.