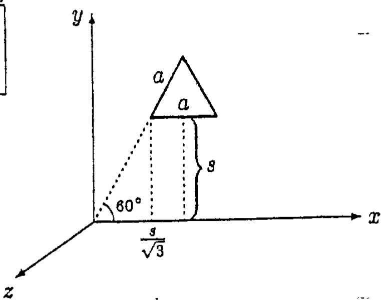
Øving 5 - Løsningsforslag

Problem 5.10

(a) The forces on the two sides cancel. At the bottom, $B = \frac{\mu_0 I}{2\pi s} \Rightarrow F = \left(\frac{\mu_0 I}{2\pi s}\right) Ia = \frac{\mu_0 I^2 a}{2\pi s}$ (up). At the top, $B = \frac{\mu_0 I}{2\pi(s+a)} \Rightarrow F = \frac{\mu_0 I^2 a}{2\pi(s+a)}$ (down). The net force is $\boxed{\frac{\mu_0 I^2 a^2}{2\pi s(s+a)}}$ (up). $\frac{a}{s}$

(b) The force on the bottom is the same as before, $\mu_0 I^2/2\pi$ (up). On the left side, $\mathbf{B} = \frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}$;
$d\mathbf{F} = I(d\mathbf{l} \times \mathbf{B}) = I(dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \times \left(\frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}\right) = \frac{\mu_0 I^2}{2\pi y}(-dx\,\hat{\mathbf{y}} + dy\,\hat{\mathbf{x}})$. But the x component cancels the corresponding term from the right side, and $F_y = -\frac{\mu_0 I^2}{2\pi}\int_{s/\sqrt{3}}^{(s/\sqrt{3}+a/2)} \frac{1}{y}\,dx$. Here $y = \sqrt{3}x$, so
$F_y = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(\frac{s/\sqrt{3}+a/2}{s/\sqrt{3}}\right) = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)$. The force on the right side is the same, so the net force on the triangle is $\boxed{\frac{\mu_0 I^2}{2\pi}\left[1 - \frac{2}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)\right]}$.

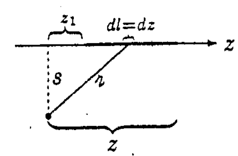
$\frac{a}{s}$



Problem 5.12

Magnetic attraction per unit length (Eqs. 5.37 and 5.13): $f_m = \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d}$.
Electric field of one wire (Eq. 2.9): $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}$. Electric repulsion per unit length on the other wire:
$f_e = \frac{1}{2\pi\epsilon_0}\frac{\lambda^2}{d}$. They balance when $\mu_0 v^2 = \frac{1}{\epsilon_0}$, or $\boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}}.}$ Putting in the numbers,
$v = \frac{1}{\sqrt{(8.85\times10^{-12})(4\pi\times10^{-7})}} = \boxed{3.00\times10^8\ \text{m/s}.}$ This is precisely the *speed of light(!)*, so in fact you could *never* get the wires going fast enough; the electric force always dominates.

Problem 5.22



$$\mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{I\,\hat{\mathbf{z}}}{r}\,dz = \frac{\mu_0 I}{4\pi}\,\hat{\mathbf{z}}\int_{z_1}^{z_2}\frac{dz}{\sqrt{z^2+s^2}}$$
$$= \frac{\mu_0 I}{4\pi}\,\hat{\mathbf{z}}\left[\ln\left(z+\sqrt{z^2+s^2}\right)\right]\Big|_{z_1}^{z_2} = \boxed{\frac{\mu_0 I}{4\pi}\ln\left[\frac{z_2+\sqrt{(z_2)^2+s^2}}{z_1+\sqrt{(z_1)^2+s^2}}\right]\hat{\mathbf{z}}}$$

$$\begin{aligned}
\mathbf{B} &= \nabla\times\mathbf{A} = -\frac{\partial A}{\partial s}\,\hat{\boldsymbol{\phi}} = -\frac{\mu_0 I}{4\pi}\left[\frac{1}{z_2+\sqrt{(z_2)^2+s^2}}\frac{s}{\sqrt{(z_2)^2+s^2}} - \frac{1}{z_1+\sqrt{(z_1)^2+s^2}}\frac{s}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}}\\
&= -\frac{\mu_0 I s}{4\pi}\left[\frac{z_2-\sqrt{(z_2)^2+s^2}}{(z_2)^2-[(z_2)^2+s^2]}\frac{1}{\sqrt{(z_2)^2+s^2}} - \frac{z_1-\sqrt{(z_1)^2+s^2}}{z_1^2-[(z_1)^2+s^2]}\frac{1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}}\\
&= -\frac{\mu_0 I s}{4\pi}\left(-\frac{1}{s^2}\right)\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - 1 - \frac{z_1}{\sqrt{(z_1)^2+s^2}} + 1\right]\hat{\boldsymbol{\phi}} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - \frac{z_1}{\sqrt{(z_1)^2+s^2}}\right]\hat{\boldsymbol{\phi}},
\end{aligned}$$

or, since $\sin\theta_1 = \frac{z_1}{\sqrt{(z_1)^2+s^2}}$ and $\sin\theta_2 = \frac{z_2}{\sqrt{(z_2)^2+s^2}}$,

$$= \boxed{\frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)\,\hat{\boldsymbol{\phi}}}\ \text{(as in Eq. 5.35).}$$