Øving 9 - Løsningsforslag

OBS! Mange eksamensoppgaver (noen med løsning), f.o.m. 1971, finnes på nettet: **http://bohr.phys.ntnu.no/AR/#SIF4060**
(faget hadde tidligere andre nummer og navn: 1971-88: Fag 71516 Elektromagnetisk teori, 1989-99: Fag 74316: Elektrisitet og magnetisme. Pensum i disse fagene kan avvike noe, men ikke svært mye, fra vårt)

Problem 9.10

$P = \frac{I}{c} = \frac{1.3\times10^3}{3.0\times10^8} = \boxed{4.3\times10^{-6}\,\text{N/m}^2.}$ For a perfect reflector the pressure is twice as great: $\boxed{8.6\times10^{-6}\,\text{N/m}^2.}$ Atmospheric pressure is $1.03\times10^5\,\text{N/m}^2$, so the pressure of light on a reflector is $(8.6\times10^{-6})/(1.03\times10^5) = \boxed{8.3\times10^{-11}\text{ atmospheres.}}$

Problem 9.11

$$\begin{aligned}\langle fg\rangle &= \frac{1}{T}\int_0^T a\cos(\mathbf{k}\cdot\mathbf{r}-\omega t+\delta_a)b\cos(\mathbf{k}\cdot\mathbf{r}-\omega t+\delta_b)\,dt\\ &= \frac{ab}{2T}\int_0^T[\cos(2\mathbf{k}\cdot\mathbf{r}-2\omega t+\delta_a+\delta_b)+\cos(\delta_a-\delta_b)]\,dt = \frac{ab}{2T}\cos(\delta_a-\delta_b)T = \frac{1}{2}ab\cos(\delta_a-\delta_b).\end{aligned}$$

Meanwhile, in the complex notation: $\tilde{f} = \tilde{a}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, $\tilde{g} = \tilde{b}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, where $\tilde{a} = ae^{i\delta_a}$, $\tilde{b} = be^{i\delta_b}$. So $\frac{1}{2}\tilde{f}\tilde{g}^* = \frac{1}{2}\tilde{a}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\tilde{b}^*e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \frac{1}{2}\tilde{a}\tilde{b}^* = \frac{1}{2}abe^{i(\delta_a-\delta_b)}$, $\text{Re}\left(\frac{1}{2}\tilde{f}\tilde{g}^*\right) = \frac{1}{2}ab\cos(\delta_a-\delta_b) = \langle fg\rangle$. qed

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Problem 9.12

$$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right).$$

With the fields in Eq. 9.48, **E** has only an x component, and **B** only a y component. So all the "off-diagonal" ($i \neq j$) terms are zero. As for the "diagonal" elements:

$$\begin{aligned}T_{xx} &= \epsilon_0\left(E_xE_x - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\frac{1}{2}B^2\right) = \frac{1}{2}\left(\epsilon_0E^2 - \frac{1}{\mu_0}B^2\right) = 0.\\ T_{yy} &= \epsilon_0\left(-\frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_yB_y - \frac{1}{2}B^2\right) = \frac{1}{2}\left(-\epsilon_0E^2 + \frac{1}{\mu_0}B^2\right) = 0.\\ T_{zz} &= \epsilon_0\left(-\frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\frac{1}{2}B^2\right) = -u.\end{aligned}$$

So $\boxed{T_{zz} = -\epsilon_0E_0^2\cos^2(kz-\omega t+\delta)}$ (all other elements zero).

The momentum of these fields is in the z direction, and it is being *transported* in the z direction, so *yes*, it does make sense that T_{zz} should be the only nonzero element in T_{ij}. According to Sect. 8.2.3, $-\overleftrightarrow{T}\cdot d\mathbf{a}$ is the rate at which momentum crosses an area $d\mathbf{a}$. Here we have *no* momentum crossing areas oriented in the x or y direction; the momentum per unit time per unit area flowing across a surface oriented in the z direction is $-T_{zz} = u = \wp c$ (Eq. 9.59), so $\Delta p = \wp cA\Delta t$, and hence $\Delta p/\Delta t = \wp cA =$ momentum per unit time crossing area A. Evidently $\boxed{\text{momentum flux density = energy density.}}$ ✓