Øving 12 - Løsningsforslag

Problem 10.11

In this case $\dot{\rho}(\mathbf{r},t) = \dot{\rho}(\mathbf{r},0)$ and $\dot{\mathbf{J}}(\mathbf{r},t) = 0$, so Eq. 10.29 $\Rightarrow$

$$\begin{aligned}
\mathbf{E}(\mathbf{r},t) &= \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(\mathbf{r}',0)+\dot{\rho}(\mathbf{r}',0)t_r}{\mathscr{r}^2}+\frac{\dot{\rho}(\mathbf{r}',0)}{c\mathscr{r}}\right]\hat{\mathscr{r}}\,d\tau', \text{ but } t_r = t-\frac{\mathscr{r}}{c} \text{ (Eq. 10.18), so} \\
&= \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(\mathbf{r}',0)+\dot{\rho}(\mathbf{r}',0)t}{\mathscr{r}^2}-\frac{\dot{\rho}(\mathbf{r}',0)(\mathscr{r}/c)}{\mathscr{r}^2}+\frac{\dot{\rho}(\mathbf{r}',0)}{c\mathscr{r}}\right]\hat{\mathscr{r}}\,d\tau' = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}',t)}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau'. \quad \text{qed}
\end{aligned}$$

Problem 10.12

In this approximation we're dropping the higher derivatives of $\mathbf{J}$, so $\dot{\mathbf{J}}(t_r) = \dot{\mathbf{J}}(t)$, and Eq. 10.31 $\Rightarrow$

$$\begin{aligned}
\mathbf{B}(\mathbf{r},t) &= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}^2}\left[\mathbf{J}(\mathbf{r}',t)+(t_r-t)\dot{\mathbf{J}}(\mathbf{r}',t)+\frac{\mathscr{r}}{c}\dot{\mathbf{J}}(\mathbf{r}',t)\right]\times\hat{\mathscr{r}}\,d\tau', \text{ but } t_r - t = -\frac{\mathscr{r}}{c} \text{ (Eq. 10.18), so} \\
&= \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'. \quad \text{qed}
\end{aligned}$$