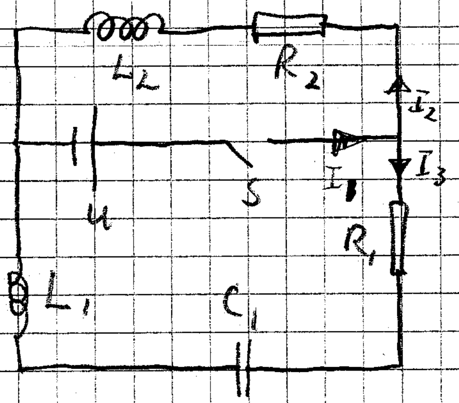
①

a) $I_3 = 0$ $\quad I_1 = I_2 = \frac{U}{R_2} = \frac{12}{5} A = 2.4 A$

b) Spenningen U står over C_1

$\Rightarrow Q = CU = 5 \cdot 10^{-6} \cdot 12\, C = 60 \mu C$



c) Krets med $R = R_1 + R_2 = (4+5)\,\Omega = 9\,\Omega$

og $L = L_1 + L_2 = (50+100)\,mH = 150\,mH$

$$\frac{Q}{C_1} - RI - L\frac{dI}{dt} = 0$$

$$\frac{Q}{C_1} - R\left(-\frac{dQ}{dt}\right) - L\left(-\frac{d^2Q}{dt^2}\right) = 0$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC_1} = 0$$

$$Q = Q_0 e^{-\alpha t}\cos(\omega t + \varphi)$$

$$\alpha = \frac{R}{2L} \qquad \omega = \sqrt{\frac{1}{LC_1} - \frac{R^2}{4L^2}}$$

Oscillerende ladning Q

$I = -\frac{dQ}{dt}$ (ut pkt e)

$t = 0$: $Q = Q_0 \Rightarrow \varphi = 0$

Oscillasjon hvis $\frac{R^2}{4L^2} < \frac{1}{LC} \Rightarrow$ hvis $R^2 < \frac{4L}{C}$

$R^2 = 9^2\,\Omega^2 = 81\,\Omega^2$

$\frac{4L}{C_1} = \frac{4 \cdot 150 \cdot 10^{-3}}{5 \cdot 10^{-6}}\,\Omega^2 = 120000\,\Omega^2 \quad \Rightarrow R^2 \ll \frac{4L}{C_1}$

$\Rightarrow$ Krets med oscillerende ladning Q og oscillerende strøm $I = -\frac{dQ}{dt}$

d) $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\frac{1}{150 \cdot 10^{-3}\, 5 \cdot 10^{-6}} - \frac{9^2}{4 \cdot 150^2 \cdot 10^{-6}}}\ s^{-1}$

$= \sqrt{10^6 \cdot 1{,}333 - 10^6 \cdot 0.00001} = 1.15469 \cdot 10^3\ s^{-1}$

$f = \frac{\omega}{2\pi} = \frac{1.15447}{2\pi} \cdot 10^3\ Hz = 183{,}8\ Hz$

e) $Q = Q_0 e^{-\alpha t}\cos\omega t = Q_0 e^{-\frac{R}{2L}t}\cos\omega t$

$I = -\frac{dQ}{dt} = -\left[Q_0\left(-\frac{R}{2L}\right)e^{-\frac{R}{2L}t}\cos\omega t - Q_0 e^{-\frac{R}{2L}}\omega\sin\omega t\right]$

$= Q_0 e^{-\frac{R}{2L}t}\left(\frac{R}{2L}\cos\omega t + \omega\sin\omega t\right)$

Men $\frac{R}{2L} = \frac{9}{2\cdot 150\cdot 10^{-3}}\,s^{-1} = 30\,s^{-1}$ mens $\omega = 1155\,s^{-1}$ (pkt d))

$\Rightarrow \frac{R}{2L} << \omega$ slik at vi med god tilnærmelse kan sette $\omega \approx \frac{1}{\sqrt{LC_1}}$ (se også alternativ løsning)

$\Rightarrow I \approx Q_0 e^{-\frac{R}{2L}t}\cdot\frac{1}{\sqrt{LC}}\sin\omega t = I_0 e^{-\frac{R}{2L}t}\sin\omega t$

$I_0 \approx \frac{Q_0}{\sqrt{LC_1}} = \frac{60\cdot 10^{-6}}{\sqrt{150\cdot 10^{-3}\cdot 5\cdot 10^{-6}}}\,A = \underline{0{,}0693\,A}$

f) Amplituden er halvert når

$e^{-\alpha t} = e^{-\frac{R}{2L}t} = \frac{1}{2} \Longrightarrow -\frac{R}{2L}t = -\ln 2$

$t = \frac{2L}{R}\ln 2 = \frac{2\cdot 150\cdot 10^{-3}}{9}\ln 2\ s \qquad = 0{,}0231\,s$

Perioden $T = \frac{1}{f} = \frac{1}{183.8}\,s = 0.00544\,s$

Antall perioder før halvering av amplituden:

$\frac{t}{T} = \frac{0.0231}{0.00544}$ svingninger $= \underline{4.25 \text{ svingninger}}$

g) Kritisk demping eller overdemping for

$R^2 \geq \frac{4L}{C_1}$, dvs for $R \geq \sqrt{\frac{4\cdot 150\cdot 10^{-3}}{5\cdot 10^{-6}}}\,\Omega = 346\,\Omega$

dvs for $R_1 \geq 346\,\Omega - R_2 = 346\,\Omega - 5\,\Omega = \underline{341\,\Omega}$

Alternativt for spørsmål c):

$$\frac{Q_0 R}{2L}\cos\omega t + Q_0\omega\sin\omega t = A\sin(\omega t+\varphi)$$

$$= A\sin\omega t\cos\varphi + A\cos\omega t\sin\varphi$$

$$A\sin\varphi = \frac{Q_0 R}{2L}$$

$$A\cos\varphi = Q_0\omega$$

$$\tan\varphi = \frac{R}{2L\omega} = \frac{9}{2\cdot 150\cdot 10^{-3}\cdot 1155} = 0{,}02597$$

$\varphi = \underline{1{,}49^\circ} = \underline{0.026}$ radianer

$$A = \frac{Q_0 R}{2L\sin 1{,}49^\circ} = \frac{60\cdot 10^{-6}\cdot 9\ \text{A}}{2\cdot 150\cdot 10^{-3}\sin 1{,}49} = 69{,}2\cdot 10^{-3}\,\text{A} = \underline{0{,}0692\ \text{A}}$$

Som viser at approksimasjonen i c) er god.

$$\Rightarrow I = A\sin(\omega t+\varphi) = \underline{0{,}0692\sin(\omega t+0{,}026)\ \text{A}}$$