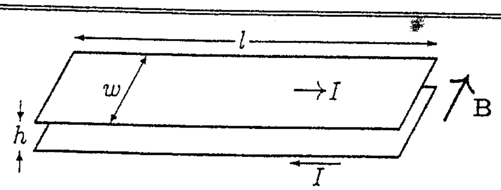
Øving 7 - Løsningsforslag

Problem 7.3

(a) $I = \int \mathbf{J} \cdot d\mathbf{a}$, where the integral is taken over a surface enclosing the positively charged conductor. But $\mathbf{J} = \sigma \mathbf{E}$, and Gauss's law says $\int \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q$, so $I = \sigma \int \mathbf{E} \cdot d\mathbf{a} = \frac{\sigma}{\epsilon_0} Q$. But $Q = CV$, and $V = IR$, so $I = \frac{\sigma}{\epsilon_0} CIR$, or $\boxed{R = \dfrac{\epsilon_0}{\sigma C}.}$ qed

(b) $Q = CV = CIR \Rightarrow \frac{dQ}{dt} = -I = -\frac{1}{RC} Q \Rightarrow \boxed{Q(t) = Q_0 e^{-t/RC}}$, or, since $V = Q/C$, $V(t) = V_0 e^{-t/RC}$. The time constant is $\tau = RC = \boxed{\epsilon_0/\sigma.}$

Problem 7.58



(a) Parallel-plate capacitor: $E = \dfrac{1}{\epsilon_0}\sigma$; $V = Eh = \dfrac{1}{\epsilon_0}\dfrac{Q}{wl}h \Rightarrow C = \dfrac{Q}{V} = \dfrac{\epsilon_0 wl}{h} \Rightarrow \boxed{C = \dfrac{\epsilon_0 w}{h}.}$

(b) $B = \mu_0 K = \mu_0 \dfrac{I}{w}$; $\Phi = Bhl = \dfrac{\mu_0 I}{w} hl = LI \Rightarrow L = \dfrac{\mu_0 h}{w} l \Rightarrow \boxed{\mathcal{L} = \dfrac{\mu_0 h}{w}.}$

(c) $\boxed{\mathcal{C}\mathcal{L} = \mu_0 \epsilon_0} = (4\pi \times 10^{-7})(8.85 \times 10^{-12}) = \boxed{1.112 \times 10^{-17}\,\mathrm{s^2/m^2}.}$
(Propagation speed $1/\sqrt{\mathcal{L}\mathcal{C}} = 1/\sqrt{\mu_0\epsilon_0} = 2.999 \times 10^8\,\mathrm{m/s} = c$.)

(d) $D = \sigma$, $E = D/\epsilon = \sigma/\epsilon$, so just replace ϵ_0 by ϵ;
$H = K$, $B = \mu H = \mu K$, so just replace μ_0 by μ. $\Big\}$ $\boxed{\mathcal{L}\mathcal{C} = \epsilon\mu;}$ $\boxed{v = 1/\sqrt{\epsilon\mu}.}$